RIGA 2021 - BOOK OF ABSTRACTS

- according to the program -

Prescribing curvature to spherical helicoidal surfaces

Ildefonso Castro[1], Ildefonso Castro-Infantes[2] and Jesús Castro-Infantes[3]

[1] Department of Mathematics, University of Jaén, Spain
[2] Department of Mathematics, University of Murcia, Spain
[3] Department of Geometry and Topology, University of Granada, Spain

We deal with helicoidal surfaces in the unit 3-sphere $\mathbb{S}^3$, i.e. surfaces invariant under the action of the helicoidal 1-parameter group of isometries given by the composition of a translation and a rotation in $\mathbb{S}^3$. This class includes the rotational ones. Spherical helicoidal flat surfaces were studied in [3]. We introduce the notion of *spherical angular momentum* of the generatrix curve of the helicoidal surface. It will play a key role since determines the geometry of the helicoidal surface joint to its pitch. Then, inspired by [1], we show that if we prescribe the mean curvature of a spherical helicoidal surface in terms of a function depending on the distance to its axis, we get a one parameter family of helicoidal surfaces with this prescribed mean curvature. As a first application, we identify the minimal spherical surfaces that play the role of classical catenoids and helicoids and describe all the minimal helicoidal surfaces in $\mathbb{S}^3$, proving that they are the *associated* surfaces ([2]) to them.

REFERENCES

[1] C. Baikoussis and T. Koufogiorgos, *Helicoidal surfaces with prescribed mean or Gaussian curvature*, J. Geom., 63 (1998), 25–29.

[2] H.B. Lawson, *Complete minimal surfaces in* $\mathbb{S}^3$, Ann. of Math., 92 (1970), 335–374.

[3] F. Manfio and J.P. dos Santos, *Helicoidal flat surfaces in the 3-sphere*, Math. Nachr., 292 (2019), 127–136.

Rigging technique on null hypersurfaces of indefinite Kaehler manifolds

Rakesh Kumar

Department of Basic and Applied Sciences,
Punjabi University, Patiala, India
rakesh_bas@pbi.ac.in

We study null hypersurfaces of indefinite Kaehler manifolds and by taking the advantages of the almost complex structure J, we select a suitable rigging ζ, call it as the $J-$rigging, on the null hypersurface. This suitable rigging enables us to build an associated Hermitian metric $\breve{g}$ on the ambient space and which is restricted into a non-degenerated metric $\widetilde{g}$ on the normalized null hypersurface (M, ζ). We derive Gauss-Weingarten type formulae for null hypersurface M of an indefinite Kaehler manifold $\overline{M}$ with a fixed closed Killing J-rigging for M. Later, we establish some relations linking the curvatures, holomorphic sectional curvatures, null sectional curvatures, Ricci curvatures, scalar curvatures etc. of the ambient manifold and normalized null

hypersurface (M,ζ). We also provide condition for the null hypersurface to be a locally product manifold.

References

[1] C. Atindogbe, M. Gutierrez and R. Hounnonkpe, *New properties on normalized null hypersurfaces*, Mediterr. J. Math., **15** (2018), 1–19.
[2] M. Barros and A. Romero, *Indefinite Kaehler manifolds*, Math. Ann., **261** (1982), 55–62.
[3] A. Bejancu and K. L. Duggal, *Lightlike submanifolds of semi-Riemannian manifolds*, Acta Appl. Math., **38** (1995), 197–215.
[4] K. L. Duggal and A. Bejancu, *Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications*, Kluwer Academic Publishers, Netherlands, (1996).
[5] K. L. Duggal and B. Sahin, *Differential Geometry of Lightlike Submanifolds*, Birkhauser Verlag AG, Berlin, (2010).
[6] M. Gutierrez and B. Olea, *Induced Riemannian structures on null hypersurfaces*, Math. Nachr., **289** (2016), 1219–1236.
[7] F. Ngakeu and H. F. Tetsing, *$\alpha-$associated metrics on rigged null hypersurfaces*, Turkish J. Math., **43** (2019), 1161–1181.
[8] B. O'Neill, *Semi-Riemannian Geometry with Applications to Relativity*, Academic Press, New York, (1983).
[9] H. F. Tetsing, F. Ngakeu and B. Olea, *Rigging technique for 1-lightlike submanifolds and preferred rigged connections*, Mediterr. J. Math., **16** (2019), 1–20.

First natural connection on almost paracontact almost paracomplex Riemannian manifolds

Hristo Manev

Medical University – Plovdiv, Faculty of Public Health,
Department of Medical Informatics, Biostatistics and e-Learning,
15-A Vasil Aprilov Blvd., Plovdiv 4002, Bulgaria

The first natural connection on almost paracontact almost paracomplex Riemannian manifolds is constructed. The object of the considerations are the main classes of the considered manifolds in which the fundamental tensor is expressed explicitly. It is obtained a relation between the studied natural connection and the Levi-Civita connection in each of these classes as well as the dependences between their respective curvature characteristics.

Hyperspheres in Euclidean and Minkowski 4-spaces as almost paracontact almost paracomplex Riemannian manifolds

Mancho Manev[1,2] and Veselina Tavkova[1]

[1]Department of Algebra and Geometry, Faculty of Mathematics and Informatics,
University of Plovdiv Paisii Hilendarski, 24, Tzar Asen St, 4000 Plovdiv, Bulgaria
[2]Department of Medical Informatics, Biostatistics and E-Learning,
Faculty of Public Health, Medical University of Plovdiv,
15A, Vasil Aprilov Blvd, 4002 Plovdiv,Bulgaria
mmanev@uni-plovdiv.bg, vtavkova@uni-plovdiv.bg

The object of our study is geometry of the so-called almost paracontact almost paracomplex Riemannian manifolds $(\mathcal{M},f,\xi,\eta,g)$. This means that $\mathcal{M}$ is an odd-dimensional real differentiable manifold, (f,ξ,η) is an almost paracontact structure and g is a Riemannian metric.

Furthermore, the restriction of the almost paracontact structure on the paracontact distribution $\mathcal{H} = \ker(\eta)$ is an almost paracomplex structure, i.e. a traceless product structure. The more popular case is when the compatible metric g with the almost paracontact structure is Riemannian, although the metric can be also indefinite.

An object of particular interest in our research is the case of the lowest dimension (which is three) of manifolds under study.

In the present work, we use two different approaches to construct an almost paracontact almost paracomplex Riemannian manifold on a hypersphere. The first case is of a hypersphere in Euclidean space $\mathbf{E}^4$ and the second is of a time-like hypersphere in pseudo-Euclidean space $\mathbf{E}^4_1$ (i.e. Minkowski space).

The purpose of this work is to study the basic geometric characteristics of the considered manifolds. The constructed manifolds are characterised with respect to their curvature properties. The obtained results provide explicit examples of the lowest dimension of the manifolds under study and will contribute to the understanding of their geometry.

Wintgen inequality for Kulkarni curvature tensor satisfying algebraic Gauss equation and its applications

Mukut Mani Tripathi

Department of Mathematics, Institute of Science
Banaras Hindu University, Varanasi 221005, India
mmtripathi66@yahoo.com

In 1979, P. Wintgen [6] proved an inequality for a surface M^2 in $\mathbb{R}^4$, namely

$$K \leq \|H\|^2 - \left|K^\perp\right|,$$

where K is the Gauss curvature, $\|H\|^2$ is the squared mean curvature and $K^\perp$ the normal curvature. The equality holds if and only if the ellipse of curvature of M^2 in E^4 is a circle. Later, for surfaces M^2 of any codimension $m \geq 2$ in real space forms $\widetilde{M}^{2+m}(c)$, B. Rouxel (1981) and I.V. Guadalupe and L. Rodriguez (1983) independently gave an extension of the Wintgen inequality as

$$K \leq \|H\|^2 - K^\perp + c.$$

In 1999, the above Wintgen inequality was conjectured by De Smet, Dillen, Verstraelen and Vrancken [2] for submanifolds in real space forms, now known as the DDVV conjecture. The DDVV conjecture was proved by Lu (2011) [5] and by Ge and Tang (2008) [3], independently. Now, corresponding inequalities have drawn interest of many geometers and they proved similar inequalities in different situations.

In this presentation, Wintgen inequality for a Kulkarni curvature tensor satisfying algebraic Gauss equation will be given as follows:

Theorem. Let (M, g) be an n-dimensional Riemannian manifold and (B, g_B) an m-dimensional Riemannian vector bundle over M with $n, m \geq 2$. Let ζ be a B-valued symmetric $(1,2)$-tensor field and T a Kulkarni curvature tensor [4] on M satisfying the algebraic Gauss equation [1, Chen, Dillen, and Verstraelen 2005]

$$T(X, Y, Z, W) = g_B(\zeta(X, W), \zeta(Y, Z)) - g_B(\zeta(X, Z), \zeta(Y, W)).$$

Then, normalized T-scalar curvature τ^T_{NOR} and normalized Wintgen curvature $\mho^\zeta_{\text{NOR}}$ of ζ satisfy

$$\tau^T_{\text{NOR}} \leq \frac{1}{n^2}\left\|\operatorname{trace}\zeta\right\|^2 - \mho^\zeta_{\text{NOR}}. \tag{1}$$

The equality case of (1) is satisfied identically if and only if, with respect to a suitable orthonormal frame $\{e_1,\dots,e_n\}$ on M and an orthonormal frame $\{e^\perp_{n+1},\dots,e^\perp_{n+m}\}$ of the Riemannian vector bundle (B,g_B), the matrices $\left(\zeta^r_{ij}\right)$ take the forms

$$\left(\zeta^{n+1}_{ij}\right) = \begin{pmatrix} a_1+b & 0 & 0 & \cdots & 0 \\ 0 & a_1-b & 0 & \cdots & 0 \\ 0 & 0 & a_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_1 \end{pmatrix},$$

$$\left(\zeta^{n+2}_{ij}\right) = \begin{pmatrix} a_2 & b & 0 & \cdots & 0 \\ b & a_2 & 0 & \cdots & 0 \\ 0 & 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_2 \end{pmatrix},$$

$$\left(\zeta^{n+3}_{ij}\right) = a_3 I_n,$$

$$\left(\zeta^{r}_{ij}\right) = 0_n, \qquad r\in\{n+4,\dots,n+m\},$$

where a_1, a_2, a_3 and b are real functions on M.

It will be explained that, applying this result a number of results can be obtained for submanifolds of Riemannian manifolds, real space forms, complex space forms, Sasakian space forms, quaternion space forms etc.

References

[1] B.-Y. Chen, F. Dillen, L. Verstraelen, *δ-invariants and their applications to centroaffine geometry*, Differential Geom. Appl. **22** (2005), no. 3, 341–354. **MR2166127 (2006i:53010)**.

[2] P.J. De Smet, F. Dillen, L. Verstraelen, L. Vrancken, *A pointwise inequality in submanifold theory*, Arch. Math. (Brno) **35** (1999), no. 2, 115-128. **MR1711669 (2000h:53072)**.

[3] J. Ge, Z. Tang, *A proof of the DDVV conjecture and its equality*, Pacific J. Math. **237** (2008), no. 1, 87-95. **MR2415209 (2009d:53080)**.

[4] R.S. Kulkarni, *Curvature and metric*, Ann. Math. (2) **91** (1970), 311-331. **MR0257932 (41 #2581)**.

[5] Z. Lu, *Normal scalar curvature conjecture and its applications*, J. Functional Anal. **261** (2011), no. 5, 1284-1308. **MR2807100 (2012f:53112)**.

[6] P. Wintgen, *Sur l'inégalité de Chen-Wilmore*, C. R. Acad. Sci. Paris Ser. A-B, **288** (1979), no. 21, A993-A995. **MR0540375 (80k:53088)**.

Riemannian submersions with differential geometry of certain curves

Tunahan Turhan[1], Gözde Özkan Tükel[2], Bayram Şahin[3]

[1]Department of Electronic Technology, Isparta University of Applied Sciences, Turkey
[2]Department of Banking and Insurance, Isparta University of Applied Sciences,Turkey
[3]Department of Mathematics, Ege University, Turkey

We study the concept of Riemaniann submersions defined between Riemannian manifolds by aid of certain curves. We firstly get a relation between curvatures of general Frenet curves lies on base manifold and corresponding Frenet curve on target manifold. Then, we present characterizations of Riemannian submersions when the corresponding curves of general Frenet curve on the total manifold have special properties. Also, we obtain certain differential equations involving elements of Riemannian submersions when a geodesic, a circle or a helix in the total manifold is transformed to a geodesic, circle or helix in the target manifold.

References

[1] T. Adachi, *Foliation on the moduli space of extrinsic circular trajectories on a complex hyperbolic space*, Topology Appl., vol. 196 (2015), 311–324.

[2] T. Adachi, M. Kameda, S. Maeda, *Geometric meaning of Sasakian space forms from the viewpoint of submanifold theory*, Kodai Math. J., vol. 33 (2010), no. 3, 383-397.

[3] T. Adachi, T. Sugiyama, *A characterization of isotropic immersions by extrinsic shapes of smooth curves*, Differential Geom. Appl., vol. 26 (2008), no.3, 307–312.

[4] F.N. Ekmekçi, K. İlarslan, *Null General Helices and Submanifolds*, Bol. Soc. Math. Mexicana, vol. 9(2003), 279-286.

[5] M. Falcitelli, S. Ianus, A.M. Pastore, *Riemannian Submersions and Related Topics*, World Scientific, River Edge, NJ, 2004.

[6] A. Gray, *Pseudo Riemannian almost product manifolds and submersions*, J. Math. Mech., vol. 16(1967), 15-737.

[7] T. Ikawa, *On Some Curves in Riemannian Geometry*, Soochow Journal of Mathematics, vol.7 (1981), 44.

[8] S. Maeda, *A Characterization of Constant Isotropic Immersions by Circles*, Archiv der Mathematik, vol. 81 (2003), no. 1, $90-95$.

[9] S. Maeda, H. Tanabe, *Totally geodesic immersions of Kähler manifolds and Kähler Frenet curves*, Math. Z., vol. 252 (2006), no. 4, 787–795.

[10] K. Nomizu, K. Yano, *On Circles and Spheres in Riemannian Geometry*, Mathematische Annalen, vol. 2010(1974), no.2, $163-170$.

[11] B. O'Neill, *The fundemantal Equations of a Submersion*, Michigon Math. J., vol. 13(1966), 459-469.

[12] B. Şahin, Riemannian Submersions, *Riemannian Maps in Hermitian Geometry and Their Applications*, Elsevier, 2017.

[13] D.W. Yoon, J.W. Lee, C.W. Lee, *Osculating curves in the Galilean 4-space*, International Journal of Pure and Applied Mathematics Vol, 100(2015), no.4, 497-506

A characterization of Riemannian submanifolds by hyperelastic curves

Gözde Özkan Tükel[1], Tunahan Turhan[2], Bayram Şahin[3]

[1]Department of Banking and Insurance, Isparta University of Applied Sciences, Turkey
[2]Department of Electronic Technology, Isparta University of Applied Sciences,Turkey
[3]Department of Mathematics, Ege University, Turkey

We investigate a characterization of submanifolds of Riemannian manifolds by means of hyperelastic curves. For this purpose, we consider hyperelastic and elastic curves under isometric immersions. We obtain this characterization of the submanifold by aid of the mean curvature

vector field H. Finally, we give some results for a Riemannian manifold with constant sectional curvature.

References

[1] M. Aquib, J.W. Lee, J. W. Vilcu, D. W. Yoon, *Classification of Casorati ideal Lagrangian submanifolds in complex space forms*, Differential Geometry and its Applications, vol.63(2019), 30-49.

[2] J. Arroyo, O.J. Garay, M. Barros, *Closed free hyperelastic curves in the hyperbolic plane and Chen-Willmore Rotational Hypersurfaces.* Israel Journal of Mathematics, vol.138 (2003), 171-187.

[3] J. Arroyo, O.J. Garay, M. Mencia, *Closed free hyperelastic curves in real space forms.* Proceeding of the XII Fall Workshop on Geometry and Physics, Coimbra, (2003) , $1 - 13$.

[4] M.E. Aydin, I. Mihai, *Wintgen inequality for statistical surfaces*, Math. Inequal. Appl. vol.22 (2019), no. 1, 123132.

[5] M. Barros, A. Ferrandez, P. Lucas, M.A. Merono, *Willmore tori and Willmore-Chen submanifolds in pseudo-Riemannian spaces*, Journal of Geometry and Physics, vol. 28 (1998), 45-46.

[6] G. Brunnett, *A new characterization of plane elastic,* Mathematical Methods in Computer Aided Design II, ed. T. Lyche and L. Schumake, Academic Press, (1992), 43-56.

[7] G. Brunnett, P. Crouch, *Elastic curves on the sphere,* Adv.Comput. Math., vol. 2 (1994), no. 1, 23-40.

[8] F. Bullo, A.D. Lewis, *Geometric Control of Mechanical Systems*, Springer, 2005.

[9] B.Y. Chen, *Total mean curvature and submanifolds of finite type.* World Scientific Pub., Singapore, 1984.

[10] O.J. Garay, *Riemannian Submanifolds Shaped by the Bending Energy and Its Allies.* Proceedings of The Sixteenth International Workshop on Diff. Geometry, vol.16(2012), $57 - 70$.

[11] R. Huang, *A Note on the p-elastica in a constant curvature manifold*, Journal of Geometry and Physics, vol. 49 (2004), 343349.

[12] Ikergeometryteam, www.ikergeometry.org, Date of Access: 25.05.2019.

[13] T. Ikawa, *On Some Curves in Riemannian Geometry, Soochow Journal of Mathematics.* Scientiae Mathematicae Japonicae, vol.7 (1981), 37-44.

[14] T.A. Ivey D.A. Singer, *Knot types, homotopies and stability of closed elastic rods*, Proceedings of the London Mathematical Society, vol.79, (1999), no. 2, 429-450.

[15] N. Kosio, *Elasticae in a Riemannian submanifold*, Osaka J. Math., vol.29, (1992), no.3 539-543.

[16] J. Langer, D.A. Singer, *The Total Squared Curvature of Closed Curves.* Journal of Differential Geometry, vol. 20 (1984), 1-22.

[17] J. Langer, D.A. Singer, *Knotted elastic curves in* R^3. Journal of the London Mathematical Society, vol. 2 (1984) no.3, 512-520.

[18] S. Maeda, *A Characterization of Constant Isotropic Immersions by Circles.* Archiv der Mathematik, vol. 81, (2003), no. 1 90-95.

[19] K. Nomizu K. Yano, *On Circles and Spheres in Riemannian Geometry.* Mathematische Annalen, vol. 210 (1974), no. 2, $163 - 170$.

[20] T. Popiel, L. Noakes, *Elastica in* $SO(3)$. J. Aust. Math. Soc., vol. 83 (2007), 105124.

[21] B. O'Neill, *Isotropic and Kahler immersions,* Canadian Journal of Mathematics, vol. 17 (1965), no. 6, $907 - 915$.

[22] B. O'Neill, *Elementary Differential Geometry.* Academic Pres., New York, 1997.

[23] G. Özkan Tükel, T. Turhan, A. Yücesan, *Hyperelastic Lie Quadratics.* Honam Mathematical Journal, vol.41 (2019), no. 2, $369 - 380$.

[24] D.A. Singer, *Lectures on elastic curves and rods.* In AIP Conference Proceedings American Institute of Physics, vol. 41(2008), no.2, $369 - 380$.

[25] K. Yano, M. Kon, *Series in Pure mathematics-Volume 3: Structures on Manifolds,* World Scientific Pub., Singapore, 1984.

Nilmanifolds and totally geodesic subalgebras of nilpotent metric Lie algebras

Ágota Figula

Department of Geometry, Institute of Mathematics, University of Debrecen

Let $\mathfrak{g}$ be a Lie algebra and G be the corresponding connected and simply connected Lie group. A metric Lie algebra $(\mathfrak{g}, \langle ., . \rangle)$ is a Lie algebra $\mathfrak{g}$ together with a Euclidean inner product $\langle ., . \rangle$ on $\mathfrak{g}$. This inner product on $\mathfrak{g}$ induces a left invariant Riemannian metric on the Lie group G. If $(\mathfrak{n}, \langle ., . \rangle)$ is a nilpotent metric Lie algebra, then the corresponding nilpotent Lie group N endowed with the left-invariant metric arising from $\langle ., . \rangle$ is a Riemannian nilmanifold. The isometry group of Riemannian nilmanifolds and the totally geodesic subalgebras of metric nilpotent Lie algebras are popular subjects for investigations (cf. [2], [3], [5], [6], [7], [8]).

In this talk I would like to discuss the classification of the isometry equivalence classes and the isometry groups of Riemannian nilmanifolds on all five dimensional simply connected non two-step nilpotent Lie groups and on all simply connected standard filiform Lie groups. This result is obtained in [4]. In this classification the metric Lie algebras which possess an orthogonal direct sum decomposition into one-dimensional subspaces play an important role. We wish to determine geodesics and flat totally geodesic subalgebras in 5-dimensional nilpotent metric Lie algebras of step > 2. This result is presented in [1]. We obtained that in the non-filiform metric Lie algebras with 1-dimensional centre the geodesic vectors and flat totally geodesic subalgebras do not depend on the choice of the inner product.

References

[1] Á. Figula, A. Al-Abayechi, *Geodesic vectors and flat totally geodesic subalgebras in nilpotent metric Lie algebras*, accepted in J. Math. Sci., 2020, pp. 16.

[2] G. Cairns, A. Hinić Galić, Yu. Nikolayevsky, *Totally geodesic subalgebras of nilpotent Lie algebras*, J. Lie Theory, 23 (2013), 1023 – 1049.

[3] P. Eberlein, *Geometry of 2-step nilpotent groups with a left invariant metric*, Ann. Sci. Ecole Norm. Sup., 4e serie, 27 (1994), 611660.

[4] Á. Figula, P. T. Nagy, *Isometry classes of simply connected nilmanifolds*, J. Goemetry and Physics, 132 (2018), 370-381.

[5] S. Homolya, O. Kowalski, *Simply connected two-step homogeneous nilmanifolds of dimension* 5, Note Math., 26 (2006), 69-77.

[6] J. Lauret, *Homogeneous Nilmanifolds of Dimension* 3 *and* 4, Geom. Dedicata, 68 (1997), 145-155.

[7] P. T. Nagy and S. Homolya, *Geodesic vectors and subalgebras in two-step nilpotent metric Lie algebras*, Adv. Geometry, 15 (2015), 121-126.

[8] E. Wilson, *Isometry groups on homogeneous nilmanifolds*, Geom. Dedicata, 12 (1982), 337- 346.

On the structure of topological loops with solvable multiplication groups

Ameer Al-Abayechi[1], Ágota Figula[2]

[1]Department of Geometry, University of Debrecen
[2]Department of Geometry, University of Debrecen

A set L with a binary operation $(x, y) \mapsto x \cdot y : L \times L \to L$ is called a loop if there exists an element $e \in L$ such that $x = e \cdot x = x \cdot e$ holds for all $x \in L$ and the equations $a \cdot y = b$ and $x \cdot a = b$ have precisely one solution, which we denote by $y = a \backslash b$ and $x = b/a$. A loop L is proper if it is not a group. The left and right translations $\lambda_a = y \mapsto a \cdot y : L \to L$ and $\rho_a : y \mapsto y \cdot a : L \to L$, $a \in L$, are permutations of L. The permutation group $Mult(L) = \langle \lambda_a, \rho_a;\ a \in L \rangle$ is called the multiplication group of L. The stabilizer of the identity element $e \in L$ in $Mult(L)$ is called the inner mapping group $Inn(L)$ of L. T. Kepka and M. Niemenmaa gave a purely group theoretical criterion for a group K to be the group $Mult(L)$ of a loop L (cf. [2]): A group K is isomorphic to the multiplication group of a loop L if and only if there exist a subgroup S such that the core of S in K is trivial and left transversals A, B to S in K such that for every $a \in A$ and $b \in B$ one has $a^{-1}b^{-1}ab \in S$ and K is generated by $A \cup B$. In this case the subgroup S is the group $Inn(L)$ of L and the transversals A and B correspond to the sets of left and right translations of L, respectively.

A loop L is called topological if L is a topological space, the binary operations $(x, y) \mapsto x \cdot y$, $(x, y) \mapsto x \backslash y$, $(x, y) \mapsto y/x : L \times L \to L$ are continuous. There is a bijection between connected topological loops L having a Lie group G topologically generated by the left translations of L and the triples (G, H, σ), where G is a connected Lie group, H is a closed subgroup of G such that the core of H in G is trivial and $\sigma : G/H \to G$ is a continuous sharply transitive section such that $\sigma(H) = 1 \in G$ and the set $\sigma(G/H)$ generates G. A section $\sigma : G/H \to G$ is called sharply transitive, if the set $\sigma(G/H)$ operates sharply transitively on G/H, i.e. for any xH and yH there exists precisely one $z \in \sigma(G/H)$ with $zxH = yH$. The loop L is defined on the homogeneous space G/H with the multiplication $xH \cdot yH = \sigma(xH)yH$ (cf. [1]).

In this talk we wish to describe the structure of the solvable Lie groups which are the multiplication groups $Mult(L)$ for three-dimensional connected topological loops L. In particular we find that the solvability of the multiplication group $Mult(L)$ of L forces that L is classically solvable. Moreover, L is congruence solvable if and only if either L has a non-discrete centre or L is an abelian extension of a normal subgroup $\mathbb{R}$ by the 2-dimensional non-abelian Lie group or by an elementary filiform loop. We determine the solvable Lie groups of dimension ≤ 6 which occur as the groups $Mult(L)$ for three-dimensional loops L and we show that these loops are centrally nilpotent of class 2.

References

[1] P. T. Nagy, K. Strambach, *Loops in Group Theory and Lie Theory*, de Gruyter Expositions in Mathematics. 35. Berlin, New York, 2002.

[2] M. Niemenmaa, T. Kepka, *On Multiplication Groups of Loops*, J. Algebra, 135 (1990), 112-122.

Branched covering surfaces - new shapes, new materials and new processes

Konrad Polthier

Freie University Berlin, Germany

The classic geometric view on smooth surfaces hardly fits to the complex and often multiscale physical surface shapes in nature and, nowadays, in industrial applications.

In this talk we will introduce a new class of multi-layered surface shapes derived from recent algorithms in geometry processing and related to classic complex analysis. Multivalued functions and differential forms naturally lead to the concept of branched covering surfaces and more generally of branched covering manifolds in the spirit of Hermann Weyl's book "The Idea of a Riemann Surface " from 1913. We will illustrate and discretize basic concepts of branched (simplicial) covering surfaces starting from complex analysis and surface theory up to their recent appearance in geometry processing algorithms and artistic mathematical designs.

Applications will touch discrete and differential surface modeling, image and geometry retargeting, optimal surfaces, and novel weaved geometry representations.

Simon's type formulas in the geometry of statistical structures

Barbara Opozda

Faculty of Mathematics and Computer Sciences UJ, ul. Łojasiewicza 6, 30-348 Cracow, Poland
barbara.opozda@im.uj.edu.pl

Let A be the cubic form of a statistical structure and τ its Tchebychev form. We present some inequalities including $\|A\|$, $\|\tau\|$ and some formulas (of Simons' type) for the Laplacian of the function $\|A\|^2$. Using them and also a maximum principle, one can prove local as well as global theorems on conjugate symmetric statistical structures. We put the emphasis on differences between the case of statistical structures on affine hypersurfaces, the one of statistical structures on Lagrangian sumbanifolds and the general case of statistical manifolds.

Wintgen inequality for surfaces in Hessian manifolds

Leila Samereh[1] and Ion Mihai[2]

[1]Department of Mathematics, Faculty of Science, Arak University, Arak, 38156-8-8349, Iran
[2]Department of Mathematics, University of Bucharest, 010014 Bucharest, Romania
l-samereh@phd.araku.ac.ir, imihai@fmi.unibuc.ro

The inequality of Wintgen is a sharp geometric inequality involving the Gauss curvature (intrinsic invariant) and the normal cur- vature and squared mean curvature (extrinsic invariants), respectively, for surfaces in the 4-dimensional Euclidean space.
In the present paper we obtain a Wintgen inequality for statistical surfaces in 4-dimensional Hessian manifolds of constant Hessian curva- ture.

References

[1] M.E. Aydin, I. Mihai, *Wintgen inequality for statistical surfaces*, Math. Ineq. Appl. **22** (2019), 123–132.
[2] P.J. De Smet, F. Dillen, L. Verstraelen, L. Vrancken, *A pointwise inequality in submanifold theory*, Arch. Math. (Brno) **35** (1999), 115–128.
[3] B. Opozda, *Bochner's technique for statistical structures*, Ann. Global Anal. Geom. **48** (2015), 357–395.
[4] B. Opozda, *A sectional curvature for statistical structures*, Linear Alg. Appl. **497** (2016), 134–161.
[5] Shima, H. *The Geometry of Hessian Structures*; World Scientific: Singapore, 2007.
[6] P. Wintgen, *Sur l'inégalité de Chen-Willmore*, C.R. Acad. Sci. Paris Sér.A–B **288** (1979), A993–A995.

Statistical manifolds on tangent bundles

Leila Nourmohammadifar and Esmaeil Peyghan

Department of Mathematics, Faculty of Science, Arak University, Arak, 38156-8-8349, Iran
nourmohammadi@gmail.com, e-peyghan@araku.ac.ir

Let $(M, g, \nabla^{(\lambda)})$ be a statistical manifold. We get the Levi-Civita connectione ${}^{CG}\widehat{\nabla}$ of TM equipped with the Cheeger-Gromoll metric ${}^{CG}g$. We calculate components of the curvature tensor ${}^{CG}\widehat{R}^{(\lambda)}$ of the Levi-Civita connection ${}^{CG}\widehat{\nabla}^{(\lambda)}$ with the Cheeger Gromoll metric ${}^{CG}g$ on the tangent bundle TM. Several illustrative examples are provided, as well.

Curvature Invariants for Statistical Submanifolds

Simona Decu (Marinescu)

Department of Applied Mathematics, Bucharest University of Economic Studies, Bucharest, Romania

We obtain optimal inequalities involving the scalar curvature (intrinsic invariant) and the δ-Casorati curvatures (extrinsic invariants) of a statistical submanifold in holomorphic statistical manifolds with constant holomorphic sectional curvature. We investigate the Casorati ideal submanifolds which characterise the totally geodesic submanifolds with respect to the Levi-Civita connection.

References

[1] M. Aydin, A. Mihai, I. Mihai, *Some inequalities on submanifolds in statistical manifolds of constant curvature*, Filomat 29 (2015),465–477.

[2] B.Y. Chen, *Pseudo-Riemannian Geometry, δ-Invariants and Applications*; World Scientific: Singapore, 2011.

[3] S. Decu, S. Haesen, L. Verstraelen, *Inequalities for the Casorati Curvature of Statistical Manifolds in Holomorphic Statistical Manifolds of Constant Holomorphic Curvature*, Mathematics, 8 (2020).

[4] H. Furuhata, I. Hasegawa, *Submanifold theory in holomorphic statistical manifolds.* In *Geometry of Cauchy-Riemann Submanifolds*; S. Dragomir, M.H. Shahid, F.R Al-Solamy, Eds.; Springer Science+Business Media Singapore: Singapore, 2016; pp. 179–214.

[5] M. Milijevic, *Totally real statistical submanifolds*, Interdiscip. Inform. Sci. 21 (2015), 87–96.

[6] L. Verstraelen, *Geometry of submanifolds I. The first Casorati curvature indicatrices*, Kragujevac J. Math. 37 (2013),5–23.

[7] G. Vilcu, *An optimal inequality for Lagrangian submanifolds in complex space forms involving Casorati curvature*, J. Math. Anal. Appl. 465 (2018),1209–1222.

A class of affine hypersurfaces with constant sectional curvature

Miroslava Antić[1], Haizhong Li[2], Luc Vrancken[3], Xianfeng Wang[4]

[1]Faculty of Mathematics, University of Belgrade, Studentski trg 16, 11000 Belgrade, Serbia
[2]Department of Mathematical Sciences, Tsinghua University, Beijing 100084, P. R. China
[3] LMI, Université Polytechnique Hauts-de-France, 59313 Valenciennes, France;
Department of Mathematics, KU Leuven, 3001 Leuven, Belgium
[4]School of Mathematical Sciences and LPMC, Nankai University, Tianjin, 300071, P. R. China

We study affine locally strongly convex hypersurfaces with constant sectional curvature in the affine space R^{n+1}. By using the Tsinghua principle, i.e. by making full use of the Codazzi equations for both the shape operator S and the difference tensor K and the Ricci identity in an indirect way, we prove a nice relation involving the eigenvalues of the shape operator and the difference tensor of the affine hypersurface. Starting from this relation, we give a classification of locally strongly convex hypersurface with constant sectional curvature whose shape operator S has at most one eigenvalue of multiplicity one.

Chen inequality for statistical submanifolds in statistical manifolds of constant curvature

Hülya Aytimur

Department of Mathematics, Balikesir University, Turkey

We obtain Chen first inequality and a Chen inequality for the $\delta(2,2)$-invariant of a statistical submanifold in statistical manifolds of constant curvature.

References

[1] B.-Y. Chen, A. Mihai, I. Mihai, *A Chen first inequality for statistical submanifolds in Hessian manifolds of constant Hessian curvature*, Results Math. **74(4)** (2019), Art. 165.
[2] A. Mihai, I. Mihai, *The $\delta\,(2,2)$ invariant on statistical submanifolds of Hessian manifolds of constant Hessian curvature*, Entropy **2020**, 22(2), Art. 164.
[3] S. Amari, *Differential-Geometrical Methods in Statistics*, Springer-Verlag, 1985.
[4] B. Opozda, *A sectional curvature for statistical structures*, Linear Alg. Appl. **497** (2016), 134-161.

Applications of the Tsinghua principle

Luc Vrancken

Université de Valenciennes, France; K.U. Leuven, Belgium

In submanifold geometry one studies properties of a submanifold M of N which are invariant under a group of transformations of N. It originates with the study of curves in $\mathbb{R}^2$ and surfaces in $\mathbb{R}^3$ which are invariant under euclidean transformations (isometries of $\mathbb{R}^n$). Later one can either look at more general ambiant spaces (provided that they have sufficiently many isometries) or look at a more general class of transformations (which is for example done in affine differential geometry)

The ambiant spaces which we will consider in this talk are

- the real space forms
- the complex projective space (or complex space forms)
- the complex quadric
- the complex hyperbolic quadric
- the nearly Kaehler $\mathbb{S}^3 \times \mathbb{S}^3$
- hypersurfaces of $\mathbb{R}^{n+1}$ with respect to the group of equiaffine transformations of $\mathbb{R}^{n+1}$.

For all of these spaces there is an explicit expression for the curvature tensor and so if one studies submanifolds of these spaces there exist as seen in any book on Riemannian geometry:

- the equation of Gauss
- the equation of Codazzi
- the equation of Ricci

together with some additional equations depending on the geometry of the ambiant space.

The questions we want to consider during this lecture is:

How does the geometry (in particular the curvature) of the submanifold determine the immersion (i.e. the way it looks in the ambiant space)

One would expect that for doing so the Gauss equation

$$< R(X,Y)Z,W > = < \tilde{R}(X,Y)Z,W > + < h(Y,Z),h(X,W) > - < h(X,Z),h(Y,W) >$$

to be a good starting point. For some questions this is indeed the case. But as this equation is nonlinear (quadratic) in the components of the second fundamental form it is not always a good starting point. For some of those questions the Tsinghua principle used by Li Haizhong, Luc Vrancken and Wang Xianfeng in 2013 at Tsinghua University is a good starting point.

However, just is true for the Gauss equation, it can not be applied for just any problem. It is necessary:

- to have an explicit expression for the curvature tensor of the submanifold
- to have a tangential version of the Codazzi equation

REFERENCES

M. Antic, J. Van der Veken, L. Vrancken, *Differential Geometry of Submanifolds: Submanifolds of Almost Complex Spaces and Almost Product Spaces*, De Gruyter Studies in Mathematics.

Surfaces with constant curvature in Euclidean space: old problems and new results

Rafael López

Department of Geometry and Topology, University of Granada, Spain

Surfaces with constant Gauss curvature and constant mean curvature play a central role in classical differential geometry of Euclidean space R^3. In order to find explicit examples of such surfaces, it is natural to impose some geometric property on the surface, such for instance, that the surface is rotational or that is ruled. This idea appears already at the beginning of the classical theory: it is enough to mention works of Euler, Meusnier, Scherk, Schwarz and Riemann. In this talk, we address two techniques that were developed in the XIXth century and that have been, in part, forgotten over in the course of time.

A first class of surfaces are the translation surfaces, which are surfaces can be locally written as the sum of two space curves. We classify all translations surfaces with constant Gauss curvature. In case of minimal translation surfaces, we give a procedure to find many examples.

The second family of surfaces are those ones that can be expressed by separation of variables $f(x) + g(y) + h(z) = 0$. We give a full classification of separable surfaces with constant Gaussian curvature and if the mean curvature is a non-zero constant, we prove that the surface is rotational.

References

[1] T. Hasanis, R. López, *Translation surfaces in Euclidean space with constant Gaussian curvature*, to appear in Comm. Anal. Geom.

[2] T. Hasanis, R. López, *Classification and construction of minimal translation surfaces in Euclidean space*, to appear in , Results in Math.

[3] T. Hasanis, R. López, *Classification of separable surfaces with constant Gaussian curvature*, to appear in Manuscripta Mathematica.

[4] R. López, O. Perdomo, *Minimal translation surfaces in Euclidean space*, J. Geometric Anal. 27 (2017), 2926–2937.

Spacetimes with different forms of energy momentum tensor

Uday Chand De

Emeritus Professor (Retd.)
Department of Pure Mathematics, University of Calcutta
35, Ballygunge Circular Road, Kolkata 700019, West Bengal, INDIA
uc_de@yahoo.com

The object of the present talk is to characterize spacetimes with different types of energy momentum tensor. At first we consider spacetimes with pseudo symmetric energy momentum tensor T. We obtain a necessary and sufficient condition for a spacetime with pseudo symmetric energy momentum tensor to be a pseudo Ricci symmetric spacetime. Next we consider the spacetimes with Codazzi type of energy momentum tensor and several interesting results are pointed out. Moreover, some results related to perfect fluid spacetimes with different forms of energy momentum tensors have been obtained. We study spacetimes with quadratic Killing energy momentum tensor T and show that a GRW spacetime with quadratic Killing energy momentum tensor is an Einstein space. Finally, we have considered general relativistic spacetimes with semisymmetric energy momentum tensor and obtained some important results.

Key Words: Perfect fluid spacetimes; Einsteins field equation; energy momentum tensor; Codazzi type tensor; GRW spacetimes.

Fractional invariants of curves in Euclidean spaces of higher dimension

Meltem Ogrenmis[1] and Mehmet Bektas[2]

[1]Department of Mathematics, Firat University, Turkey
[2]Department of Mathematics, Firat University, Turkey

Yajima, et al. [2] introduced the notion of curvature with fractional order for a given Euclidean plane curve, illustrating examples with prescribed curvature. This approach was performed in a Euclidean space of dimension 3 by Aydin et al. [1] and the notions of curvature and torsion with fractional order were defined. The main purpose of this talk is to generalize in Euclidean spaces of higher dimension the mentioned objects.

References

[1] T. Yajima, S. Oiwa, K. Yamasaki, *Geometry of curves with fractional-order tangent vector and Frenet-Serret Formulas*, Fract. Calc. Appl. Anal. vol. 21(6) (2018), 1493-1505.

[2] M.E. Aydin, M. Bektas, A.O. Ogrenmis, A. Yokus, *Differential Geometry of curves in Euclidean 3-space with fractional order*, Int. Electron. J. Geom. in press.

A new approach on singular minimal surfaces in 3-spaces

Muhittin Evren Aydin

Department of Mathematics, Firat University, Turkey

This talk is based on jointly works with Ayla Erdur and Mahmut Ergut (Department of Mathematics, Namik Kemal University, Tekirdag, Turkey). The singular minimal surfaces named by Dierkes are those surfaces minimizing energy while the minimal surfaces are the ones minimizing area (locally). We approach the surfaces in the Euclidean 3-space which belong to intersecting of the sets of singular minimal and minimal surfaces. Since those surfaces are indeed nothing but a

plane, we modify the usual condition of singular minimality by using a certain semi-symmetric metric connection. With this connection, we observe that the singular minimal surfaces which are minimal are the cylindrical translating solitons which are solutions to the mean curvature flow equation for the special variation given by a subgroup of the translations. Our approach is also performed in the Lorentz-Minkowski 3-space.

References

[1] U. Dierkes, Singular Minimal Surfaces, Geometric Analysis and Nonlinear Partial Differential Equations, pp. 177-193, Springer, Berlin (2003).
[2] E. Kocakusakli, M. Ortega, A zoo of translating solitons on a parallel lightlike direction in Minkowski 3-space, arXiv:2004.04974v1 [math.DG]
[3] R. Lopez, Invariant surfaces in Euclidean space with a log-linear density, Adv. Math. 339 (2018), 285-309.
[4] R. Lopez, Some geometric properties of translating solitons in Euclidean space, J. Geom. 109(40) (2018), 1-15.
[5] Y. Wang, Minimal translation surfaces with respect to semi-symmetric connections in $\mathbb{R}^3$ and $\mathbb{R}^3_1$, arXiv:2003.12682v1 [math.DG].

Slant submanifolds in generalized Sasakian space forms
satisfying a natural equality

Pablo Alegre[1], Joaquín Barrera[2] and Alfonso Carriazo[3]

[1]Departamento de Economía, Métodos Cuantitativos e Historia Económica,
Área de Estadística e Investigación Operativa, Universidad Pablo de Olavide, Spain
[2]Servicio de Planes de Formación. Dirección General de Formación del Profesorado e
Innovación Educativa, Consejería de Educación y Deporte. Junta de Andalucía, Spain
[3]Departamento de Geometría y Topología, Facultad de Matemáticas, Universidad de Sevilla, Spain

In this talk we will focus on slant submanifolds whose second fundamental forms satisfy the equality cases of natural inequalities between their mean curvatures and their scalar curvatures. We will give several interesting examples of these submanifolds, as well as lower and upper bounds for their Ricci curvatures. Moreover, we will also present an adapted closed form for slant submanifolds of generalized Sasakian space forms, similar to the Maslov form, and we will study under which circumstances the given closed form is also conformal. Our results have been recently published in [1] and [2].

References

[1] P. Alegre, J. Barrera, A. Carriazo, *A closed form for slant submanifolds of generalized Sasakian space forms*, Mathematics, 7(12) (2019), 1238.
[2] P. Alegre, J. Barrera, A. Carriazo, *A new class of slant submanifolds in generalized Sasakian space forms*, Mediterranean Journal of Mathematics, 17 (2020), Art. 76, 18 pp.

Covariant and Lie derivatives on real hypersurfaces
in complex projective space

Juan de Dios Pérez[1] **and David Pérez-López**[2]

[1]Departamento de Geometría y Topología and
IEMATH, Universidad de Granada, 18071 Granada, Spain
[2]Departamento de Geometría y Topología, Universidad de Granada, 18071 Granada, Spain

Let M be a real hypersurface in complex projective space $\mathbb{C}P^m$. The Khler structure (J, g) of $\mathbb{C}P^m$ induces on M an almost contact metric structure (ϕ, ξ, η, g). This structure allows us to define, for any nonnull real number k, the so called k-th generalized Tanaka-Webster connection $\hat{\nabla}^{(k)}$ on M by $\hat{\nabla}^{(k)}_X Y - \nabla_X Y = F^{(k)}_X Y = g(\phi AX, Y)\xi - \eta(Y)\phi AX - k\eta(X)\phi Y$, for any X, Y tangent to M where A is the shape operator on M. The torsion of such a connection is given by $T^{(k)}(X, Y) = T^{(k)}_X Y = F^{(k)}_X Y - F^{(k)}_Y X$. If $\mathcal{L}$ denotes the Lie derivative on M we can also define a differential operator of first order $\mathcal{L}^{(k)}$ on M by $\mathcal{L}^{(k)}_X Y - \mathcal{L}_X Y = T^{(k)}_X Y$.
If K es a tensor field of type (1,1) on M we can define two tensor fields of type (1,2) on M for any nonnull real number k: $K^{(k)}_F(X, Y) = [F^{(k)}_X, K]Y$ and $K^{(k)}_T(X, Y) = [T^{(k)}_X, K]Y$. During the last five years we have classified real hypersurfaces in $\mathbb{C}P^m$ whose corresponding tensors for the shape operator, the structure Jacobi operator R_ξ or the Lie structure operator $L_\xi = \phi A - A\phi$ vanish. Here we introduce new classification theorems for such tensors to be symmetric or skew-symmetric.

References

[1] J. D. Pérez, *Commutativity of Cho and structure Jacobi operators of a real hypersurface in a complex projective space*, Ann. Mat. Pura Appl., 194 (2015), 1781-1794.
[2] J. D. Pérez and Y. J. Suh, *Generalized Tanaka-Webster and covariant derivatives on a real hypersurface in a complex projective space*, Monatsh. Math., 177 (2015), 637-647.
[3] J. D. Pérez, *Comparing Lie derivatives on real hypersurfaces in complex projective space*, Mediterr. J. Math., 13 (2016), 2161-2169.
[4] J. D. Pérez, *Lie derivatives and structure Jacobi operator on real hypersurfaces in complex projective spaces*, Diff. Geom. Appl., 50 (2017), 1-10.
[5] J. D. Pérez and David Pérez-López, *Lie derivatives and structure Jacobi operator on real hypersurfaces in complex projective spaces II*, Diff. Geom. Appl., 73 (2020), 101685.
[6] G. Kaimakamis, K. Panagiotidou and J. D. Pérez, *Derivatives of the operator $\phi A - A\phi$ on a real hypersurface in non-flat complex space forms*, Bull. Malays. Math. Sci. Soc., 43 (2020), 267-282.

Trans-S-manifolds and their Legendre curves satisfying certain conditions

Şaban Güvenç

Balikesir University, Department of Mathematics, Balikesir, Turkey

In this talk, we consider trans-S-manifolds and define their Legendre curves as in the case of S-manifolds. We find Frenet frame fields and curvatures of these Legendre curves in order to satisfy $\nabla_T T = -qfT$, where ∇ is the Levi-Civita connection, q is a non-zero constant, T is the unit tangential vector field of the curve and f is the $(1, 1)-$type tensor field of the trans-S-manifold. We also investigate conditions for these curves to have C-parallel or C-proper mean curvature vector field in the tangent and normal bundle.

References

[1] D. E. Blair, *Riemannian Geometry of Contact and Symplectic Manifolds*, Birkhauser, Boston, (2002).
[2] C. Özgür and Ş. Güvenç, *On Biharmonic Legendre curves in S-space forms*, Turk. J. Math., 38 (2014), 454–461.
[3] P. Alegre, L. M. Fernandez and A. Prieto-Martin, *A new class of metric f-manifolds*, Carpathian J. Math., 34 (2018), 123–134.

On some properties of quasi-Einstein sequential warped product manifolds

Fatma Karaca[1] and Cihan Özgür[2]

[1] Department of Mathematics, Beykent University, İstanbul, Turkey
[2] Department of Mathematics, Balıkesir University, Balıkesir, Turkey

The sequential warped product manifold is the triple product manifold $\overline{M} = (M_1 \times_f M_2) \times_h M_3$ equipped with the metric tensor

$$\overline{g} = g_1 \oplus f^2 g_2 \oplus h^2 g_3,$$

where $f : M_1 \to (0, \infty)$ and $h : M_1 \times M_2 \to (0, \infty)$ are warping functions [2]. It is known that a (semi)-Riemannian manifold of quasi-constant curvature is a quasi-Einstein manifold [1]. In this study, we consider quasi-Einstein sequential warped product manifolds. We find the main relations for a sequential warped product manifold to be a quasi-Einstein manifold. We obtain the necessary and sufficient conditions for a sequential standard static space-time and a sequential generalized Robertson-Walker space-time to be a manifold of quasi-constant curvature.

References

[1] Chen, B.-Y., Yano, K., *Hypersurfaces of a conformally flat space*, Tensor (N.S.), 26 (1972), 318-322.
[2] De, U. C., Shenawy, S., Ünal, B., *Sequential warped products: curvature and conformal vector fields*, Filomat, 33(13) (2019), 4071-4083.
[3] Dobarro, F., Ünal, B., *Special standard static space–times*, Nonlinear Analysis: Theory, Methods & Applications, 59(5) (2004), 759-770.
[4] Sular, S., Özgür, C., *On quasi-Einstein warped products*, Annals of the Alexandru Ioan Cuza University-Mathematics, 58(2) (2012), 353-362.

When a vector field is a magnetic map?

Marian Ioan Munteanu

Alexandru Ioan Cuza University of Iasi, Department of Mathematics,
Bd. Carol I, no. 11, Iasi, 700506, Romania,
marian.ioan.munteanu@gmail.com

This talk is based on some joint papers with J. Inoguchi from the Institute of Mathematics, University of Tsukuba, Japan.

In our paper [1] we define the notion of *magnetic map* as a generalization of both magnetic curves and harmonic maps. As a vector field can be thought of as a map from the manifold to its tangent bundle and since the tangent bundle carries a natural magnetic field obtained from its almost Kählerian structure, we may ask when a vector field is a magnetic map?

Furthermore, we show that a unit vector field on an oriented Riemannian manifold is a critical point of the Landau Hall functional if and only if it is a critical point of the Dirichlet energy functional. Therefore, we provide a characterization for a unit vector field to be a magnetic map into its unit tangent sphere bundle.

References

[1] J. Inoguchi and M. I. Munteanu, *Magnetic maps*, Internat. J. Geom. Methods Mod. Phys. 11 (2014) 6, art. n 1450058.

[2] J. Inoguchi and M. I. Munteanu, *New examples of magnetic maps involving tangent bundles*, Rend. Semin. Mat. Univ. Politec. Torino 73/1 (2015) 3-4, 101–116.

[3] J. Inoguchi and M. I. Munteanu, *Magnetic vector fields: New examples*, Publ. Inst. Math. Beograd 103 (117) (2018), 91–102.

[4] J. Inoguchi and M. I. Munteanu, *Magnetic unit vector fields*, submitted.

Comparison principle, duality, and harmonicity - comparing diverse phenomena and discovering secret unity

Shihshu Walter Wei

Department of Mathematics, University of Oklahoma, Norman OK 73019, U. S. A.

Duality is a special type of symmetry that involves with "polar opposites" and their dynamical interplays. The two are not merely opposites. The more we learn about human striving, the more we see they are supplementary, complementary, integrative and inextricably bound together. Duality is very elegant, yet powerful and has a long and distinguished history going back thousands of years. It is a natural and precious phenomenon that permeates or occurs in practically all branches of mathematics, physics, engineering, logic, psychology, real life, food science, social sciences, natural sciences, medical sciences such as alternative or energy medicine, acupuncture, meditation, qigong, physical therapy, nutrition therapy, etc (cf. [W2]).

Comparing diverse phenomena and discovering secret unity, we will discuss comparison theorems in differential equations and in differential geometry and the transitions between these two fields with applications in physics and bundle-valued generalized harmonic forms on noncompact manifolds in real, complex, and Finsler geometry from the viewpoint of dualities.

References

[W1] S. W. Wei, *Growth Estimates for Generalized Harmonic Forms on Noncompact Manifolds with Geometric Applications*, Geometry of Submanifolds, 247-269, Contemp. Math., 756, Amer. Math. Soc., Providence, RI, (2020).

[WW] S.W. Wei and B.Y. Wu, *Generalized Hardy type and Caffarelli-Kohn-Nirenberg type inequalities on Finsler manifolds*, to appear in Internat. J. Math. 31 (2020), no. 13, 2050109, 27 pp.

[W2] S. W. Wei, *Dualities in Comparison Theorems and Bundle-Valued Generalized Harmonic Forms*, to appear in SCI. CHINA Math., 59 pp.

Ideas for a new generation of problems in mathematical chemistry

Fănică Cimpoeşu[1] **and Adela Mihai**[2]

[1]Institute of Physical Chemistry "Ilie Murgulescu" of the Romanian Academy
[2]Technical University of Civil Engineering Bucharest
cfanica@yahoo.com, adela.mihai@utcb.ro

The term mathematical chemistry is nowadays mostly associated with applications of graph theory in topological issues of 3D chemical structures, thought as collection of atoms as dots and bonds as lines. We propose here new issues coming from the side of nowadays computational chemistry, which implies application of quantum physics to concrete problems. For instance, possible challenges can be found in examining, with ancillary tools of state-of-the art geometry, the so-called potential energy surfaces of certain specific molecular structure prototypes.

REFERENCES

[1] F. Cimpoeşu, M. Ferbinţeanu, I. Humelnicu, A. Mihai, *The symmetry blueprints of the molecular edifices*, Symmetry: Culture and Science **19 (4)** (2008), 225-432.

[2] M. Putz, F. Cimpoeşu, M. Ferbinţeanu, *Structural Chemistry*, Springer, 2018.

On the existence of convex functions on non-compact Finsler manifolds

Sorin V. Sabau

Department of Biological Sciences, Tokai University, Sapporo Campus, Japan

We show that a non-compact (forward) complete Finsler manifold whose Holmes-Thompson volume is finite admits no non-trivial convex functions. We apply this result to some Finsler manifolds whose Busemann function is convex.

References

[1] S. T. Yau, *Non-existence of Continuous Convex Functions on Certain Riemannian manifolds*, Math. Ann. 207 (1974), 269270.

[2] S. V. Sabau, K. Shiohama, *Topology of complete Finsler manifolds admitting convex functions*, 2015, Pacific Journal of Mathematics, Vol. 276 (2015), No. 2, 459481.

[3] S. V. Sabau and P. Chansangiam, *On the existence of convex functions on Finsler manifolds*, Balkan Journal of Geometry and Its Applications, Vol.24, No.1, 2019, pp. 93-103.

Cosmological Finsler spacetimes

Nicoleta Voicu

Transilvania University of Braşov, Romania

This is a joint work with Manuel Hohmann and Christian Pfeifer.

Applying the cosmological principle to Finsler spacetimes, we identify the Lie algebra of symmetry generators of spatially homogeneous and isotropic Finsler geometries, thus generalizing Friedmann-Lemaître-Robertson-Walker geometry. In particular, we determine the most general spatially homogeneous and isotropic Berwald spacetimes, which are Finsler spacetimes that can be regarded as closest to pseudo-Riemannian geometry.

Hom-symmetric spaces and Hom-Jordan Hom-symmetric spaces

Sami Chouaibi[1] **and Esmaeil Peyghan**[2]

[1]Department of Mathematics, Faculty of Sciences, Sfax University, Sfax, Tunisia
[2] Department of Mathematics, Faculty of Science, Arak University, Arak, 38156-8-8349, Iran

In this paper, we introduce and study the notions of Hom-reflection space and Hom-symmetric space. We provide some examples of Hom-reflection space (resp. Hom-symmetric space) by using ordinary reflection (resp. symmetric) spaces. Also, we associate a Hom-reflection (resp. Hom-symmetric) space to a Hom-Lie group. Finally, before showing that there is a relationship between Hom-Jordan algebras and Hom-symmetric spaces, we first provide some properties of a Hom-Jordan algebra.

Finding geodesics on surfaces using Taylor expansion of exponential map

Esa Sharahi

Arak University, Iran

Our aim is to construct a numerical algorithm using Taylor expansion of exponential map to find geodesic joining two points on a 2-dimensional surface for which a Riemannian metric is defined. This approach is generic and can be emulated in path finding for some problems that a cost function is defined.

Keywords: Euclidean space, exponential map, geodesic, navigation problem, Riemannian manifold, Taylor expansion.

On the geometry of a Randers cylinder

Rattanasak Hama[1] **and Sorin V. Sabau**[2]

[1]Prince of Songkla University, Surat Thani Campus, Thailand
[2]Tokai University, Sapporo Campus, Japan

We will present some results from the geometry of a Randers cylinder of revolution. The cylinder of revolution, in the Riemannian case, was studied by P. Chitsakul [1], [2]. On the other hand, Randers rotational metrics, constructed from the Zermelo's navigation process by the navigation data $W = (0, B)$, where B is constant, on surfaces of revolution homeomorphic to $\mathbf{R}^2$ and $\mathbf{S}^2$, were studied in [3] and [4], respectively. In the present talk we will consider Randers metric obtained from the navigation data (h, W), where $W = (A, B)$, where A, B are constants, on a surface of revolution homeomorphic to a cylinder.

References

[1] P. Chitsakul, *The structure theorem for the cut locus of a certain class of cylinders of revolution I.* Tokyo J. Math. 37 (2014), no. 2, 473–484.

[2] P. Chitsakul, *The structure theorem for the cut locus of a certain class of cylinders of revolution II.* Tokyo J. Math. 38 (2015), no. 1, 239–248.

[3] R. Hama, P. Chitsakul, S. V. Sabau, *Corrigenda: the geometry of a Randers rotational surface.* Publ. Math. Debrecen 88 (2016), no. 3-4, 517–519.

[4] R. Hama, J. Kasemsuwan, S. V. Sabau, *The cut locus of a Randers rotational 2-sphere of revolution.* Publ. Math. Debrecen 93 (2018), no. 3-4, 387–412.

Ricci soliton equation with applications to Zener schematics

Veturia Chiroiu[1], Ligia Munteanu[1] and Polidor Bratu[1,2]

[1]Department of Deformable Media and Ultrasonics,
Institute of Solid Mechanics of Romanian Academy, Bucharest
[2]Research Institute for Construction Equipment and Technologies (ICECON), Bucharest

A smooth vector field on a Riemannian manifold is a Ricci soliton if it satisfies the Ricci soliton equation. The Ricci solitons are obtained in this paper by the cnoidal theory as fixed points of the Ricci flow projected from the metrics space to its diffeomorphisms. Ricci solitons via the Bäcklund transformation associated to the pseudospherical reduction of the rheological Zener schematics are also reported.

References

[1] B.-Y. Chen, *A Survey on Ricci Solitons on Riemannian Submanifolds*, Contemporary Mathematics, vol. 674 (2016).

[2] L. Munteanu, V. Chiroiu, P. Bratu, *On the Ricci soliton equation-Part I. Cnoidal method*, Romanian Journal of Mechanics, 5(2) (2020), 22-42.

[3] P. Bratu, C. Dobrescu, *Dynamic Response of Zener-Modelled Linearly Viscoelastic Systems under Harmonic Excitation*, Symmetry (2019), 1-13.

[4] L. Munteanu, St. Donescu, *Introduction to Soliton Theory: Applications to Mechanics*, Book Series Fundamental Theories of Physics, 143, Kluwer Academic Publishers, Dordrecht, Boston (2004).

Ricci-like solitons with arbitrary potential and gradient almost Ricci-like solitons on Sasaki-like almost contact B-metric manifolds

Mancho Manev

University of Plovdiv Paisii Hilendarski, Faculty of Mathematics and Informatics,
Department of Algebra and Geometry, 24 Tzar Asen St., Plovdiv 4000, Bulgaria
and
Medical University of Plovdiv, Faculty of Public Health,
Department of Medical Informatics, Biostatistics and E-Learning,
15A Vasil Aprilov Blvd., Plovdiv 4002, Bulgaria
mmanev@uni-plovdiv.bg

In [1], the author introduced and began the study of a generalization of Ricci soliton that is compatible with the almost contact B-metric structure and its potential is the Reeb vector field ξ. There, these objects are studied on some important kinds of manifolds under consideration: Einstein-like, Sasaki-like and having a torse-forming potential ξ. The notions *Einstein-like* and *Sasaki-like* are defined as generalizations of their classic counterparts using all available metric tensors g, $\tilde{g}$ and $\eta \otimes \eta$. Then, explicit examples of Lie groups with the studied structure in dimension 3 and 5 are commented in relation with the proven assertions.

In [2], the author continues the study of Ricci-like solitons on Sasaki-like almost contact B-metric manifolds. Cases are considered in which the potential of the Ricci-like soliton is ξ or pointwise collinear to it, i.e. vertical. In the former case, the properties for a parallel or recurrent Ricci tensor are studied. In the latter case, it is shown that the potential of the considered Ricci-like soliton has a constant length and the manifold is η-Einstein. Other curvature conditions are also found, which imply that the main metric is Einstein. After that, some results are obtained for a parallel symmetric second-order covariant tensor on the manifolds under study. Finally, an explicit example of dimension 5 is given and some of the results are illustrated.

In the present work, the author explores Ricci-like solitons with arbitrary potential on Sasaki-like almost contact B-metric manifolds. The soliton under study is characterized and proved that its Ricci tensor is equal to the vertical component of both B-metrics g and $\tilde{g}$ multiplied by a constant. Thus, the scalar curvatures with respect to both B-metrics are equal and constant. In the 3-dimensional case, it is found that the special sectional curvatures with respect to the structure are constant. Gradient almost Ricci-like solitons on Sasaki-like almost contact B-metric manifolds are proved to have constant soliton coefficients. Explicit examples are provided of Lie groups as manifolds of dimensions 3 and 5 equipped with the structures under study.

References

[1] M. Manev, *Ricci-like solitons on almost contact B-metric manifolds*, J. Geom. Phys., vol. 154 (2020), 103734.
[2] M. Manev, *Ricci-like solitons with vertical potential on Sasaki-like almost contact B-metric manifolds*, Results Math., vol. 75 (2020), 136.

On Osserman and two-root manifolds

Vladica Andrejić

Faculty of Mathematics, University of Belgrade

The local isometries of a locally two-point homogeneous spaces act transitively on the sphere bundle of unit tangent vectors, and therefore fix the characteristic polynomial of the Jacobi operator there. In this way we get a generalization of locally two-point homogeneous Riemannian manifolds, called the (globally) Osserman manifolds, in which the characteristic polynomial of a Jacobi operator $\mathcal{J}_X$ is independent of X from the unit tangent bundle. The question of whether the converse is true (every Osserman manifold is locally two-point homogeneous) is known as the Osserman conjecture. Nikolayevsky established the affirmative answer in all cases, except the manifolds of dimension 16 whose reduced Jacobi operator has an eigenvalue of multiplicity 7 or 8.

We introduce k-root manifolds in which the reduced Jacobi operator has exactly k eigenvalues. We investigate one-root and two-root manifolds as another generalization of locally two-point homogeneous spaces. It is well known that a connected one-root Riemannian manifold is a space of constant sectional curvature. We present the following recent results from [1]. There is no two-root Riemannian manifold of odd dimension. There are no connected two-root Riemannian manifolds of twice an odd dimension other than those that are globally Osserman.

References

[1] V. Andrejić, *Two-root Riemannian Manifolds*, `arXiv:2009.12834 [math.DG]`, 2020.

Generalized quasi-Einstein normal metric contact pairs

İnan Ünal

Department of Computer Engineering, University of Munzur, Turkey

A normal metric contact pair manifold M, is a $(2p+2q+2)$-dimensional differentiable manifold with two 1-forms α_1, α_2 and a metric almost contact structure which is normal. M is called a generalized quasi-Einstein manifold if we have

$$Ric(X_1, X_2) = \lambda g(X_1, X_2) + \beta\alpha_1(X_1)\alpha_1(X_2) + \mu\alpha_2(X_1)\alpha_2(X_2)$$

for functions λ, β, μ on M and all $X_1, X_2 \in \Gamma(TM)$. In this talk, we present some properties of generalized quasi-Einstein normal metric contact pair manifolds. In addition, we examine normal metric contact pair manifolds which is a space of generalized quasi-constant curvature. Also, we give some results with certain curvature conditions.

References

[1] Bande, G., Hadjar, A. *Contact pairs.* Tohoku Mathematical Journal 2005; Second Series:57(2): 247-260. doi:10.2748/tmj/1119888338

[2] Bande, G., Hadjar, A. *Contact pair structures and associated metrics.* Differential Geometry - Proceedings of the $VIII$ International Colloquium 2009; (pp. 266-275).

[3] Bande, G., Hadjar, A. *On normal contact pairs.* International Journal of Mathematics 2010; 21(06): 737-754. doi: 10.1142/S0129167X10006197

[4] Bande, G., and Blair, D. E. *Symmetry in the geometry of metric contact pairs.* Mathematische Nachrichten 2013; 286(17-18): 1701-1709. doi:10.1002/mana.201100300

[5] Bande, G., Blair, D. E., Hadjar, A. *Bochner and conformal flatness of normal metric contact pairs.* Annals of Global Analysis and Geometry 2015; 48(1): 47-56. doi:10.1007/s10455-015-9456-2

[6] Blair, D. E., Ludden, G. D., and Yano, K. *Geometry of complex manifolds similar to the Calabi-Eckmann manifolds.* Journal of Differential Geometry 1974; 9(2),: 263-274.

[7] Chaki, M.C.: *On Generalized quasi-Einstein manifold.* Publ. Math. Debrecen 2001; 58: 638691

[8] Chen, B. Y., and Yano, K. *Hypersurfaces of a conformally flat space.* Tensor (NS) 1972; 26: 318-322.

[9] De, U.C., Ghosh, G.C. *On quasi Einstein manifolds.* Periodica Mathematica Hungarica 48, 223231 (2004) https://doi.org/10.1023/B:MAHU.0000038977.94711.ab

[10] De, U. C., and Ghosh, G. C. *On generalized quasi Einstein manifolds.* Kyungpook mathematical journal 2004; 44(4): 607-607.

[11] De, U. C., and Mallick, S. *On the existence of generalized quasi-Einstein manifolds.* Arch. Math.(Brno) 2011; 47(4): 279-291.

[12] De, U. C., and Shenawy, S. *Generalized quasi-Einstein GRW space-times.* International Journal of Geometric Methods in Modern Physics 2019; 16(08): 1950124. doi: 10.1142/S021988781950124X

[13] Mallick, S., and De, U. C. *On a Class of Generalized quasi-Einstein Manifolds with Applications to Relativity.* Acta Universitatis Palackianae Olomucensis. Facultas Rerum Naturalium. Mathematica 2016; 55(2): 111-127.

[14] Pokhariyal, G. P. and Mishra, R. S. *Curvature tensors and their relativistics significance*, Yokohama Mathematical Journal 1970; vol. 18: pp. 105108.

[15] Prakasha, D. G., Fernandez, L. M., and Mirji, K. *The $\mathcal{M}$-projective curvature tensor field on generalized (κ, μ)-paracontact metric manifolds.* Georgian Mathematical Journal 2020; 27(1): 141-147. doi:10.1515/gmj-2017-0054.

[16] Zengin, F. Ö. On m-projectively flat LP-Sasakian manifolds. Ukrainian Mathematical Journal 2014; 65(11): 1725-1732.

Certain submanifolds of spheres

Mirjana Djorić

University of Belgrade, Faculty of Mathematics

On this occasion we will present several results about (contact) CR-submanifolds of six and seven-dimensional spheres. There is a nearly Kähler almost complex structure J on the six-dimensional unit sphere, defined by the multiplication of Cayley numbers. A submanifold of a manifold with an almost complex structure is CR, if it has a holomorphic distribution such that its orthogonal complement in the tangent space is a totally real distribution. On the other hand, as the seven-dimensional unit sphere has the remarkable property of being a Sasakian manifold, with the almost contact metric structure (φ, ξ, η, g), we study its contact CR-submanifolds, namely those that carry a φ-invariant distribution such that its orthogonal complement in the tangent space is φ-anti-invariant.

References

[1] B.-Y. Chen, *Pseudo-Riemannian Geometry, δ-invariants and Applications*, Word Scientific, Hackensack, NJ, (2011).

[2] M. Djorić, L. Vrancken, *Three dimensional minimal CR submanifolds in S^6 satisfying Chen's equality*, J. Geom. Phys., vol. 56(11) (2006), 2279-2288.

[3] M. Antić, M. Djorić, L. Vrancken, *Four dimensional minimal CR submanifolds in S^6 satisfying Chen's equality*, Differ. Geom. Appl., vol. 25(3) (2007), 290-298.

[4] M. Djorić, M.I. Munteanu, L. Vrancken, *Four-dimensional contact CR-submanifolds in $S^7(1)$*, Math. Nachr., vol. 290(16) (2017), 2585-2596.

[5] M. Djorić, M.I. Munteanu, *On certain contact CR-submanifolds in $\mathbb{S}^7$*, Contemporary Mathematics, AMS, vol. 756 (2020), 111-120.

[6] M. Djorić, M.I. Munteanu, *Five-dimensional contact CR-submanifolds in $\mathbb{S}^7(1)$*, Mathematics, section Algebra and Geometry, special issue Riemannian geometry of submanifolds, 2020, 8, 1278, 23 pages.

Invariants on locally conformal Kähler manifolds

Gabriel-Florin Macsim[1] and Adela Mihai[2]

[1]Romania, [2]Technical University of Civil Engineering Bucharest
gabi_macsim@yahoo.com, adela.mihai@utcb.ro

We define a locally conformal Kähler Riemannian invariant δ_k and establish an inequality involving this invariant. We also characterize the equality case.

A geometric interpretation of some relations between integer sequences

Radu-Ioan Mihai

Student, Faculty of Mathematics and Computer Science, University of Bucharest
Str. Academiei 14, 010014, Bucharest, Romania
radu.mihai4@s.unibuc.ro, rimihai2001@gmail.com

In this talk, I will present a geometric interpretation of a relation between two well-known integer sequences. New research ideas will be given.

REFERENCES

[1] J. Gielis, *Inventing the Circle. The Geometry of Nature*, Geniaal, Antwerpen, 2003

[2] R.-I. Mihai, *A geometric approach to Fibonacci and Lucas sequences*, Parabola **56 (1)** (2020), UNSW, Sydney, Australia

Lamé curves in bamboo

Johan Gielis[1] and Peijian Shi[2]

[1]Department of Biosciences Engineering, University of Antwerp, B-2020 Antwerp, Belgium
[2]College of Biology and the Environment, Nanjing Forestry University, Nanjing 210037, China

At the age of 21 Gabriel Lamé (1795-1860) published a small booklet on geometrical methods [1]. In particular, to apply geometry to crystallography, he introduced supercircles and superellipses, a subset of what are now known as Lamé curves. In the second half of the 20th century these were applied in architecture and design, but only round 1993 the aspect of natural shapes was taken up with the application of these curves to model cross section of culms of square bamboos. A generalization of Lamé curves to Gielis curves enlarges the scope of a unified geometric description to many more natural shapes. In the past five years superellipses have been used extensively to model bamboo meristems, culms and leaves, annual rings and torsion in trees, and on leaves of various other plants [2]. Over 40000 different specimens were studied and in general, two numbers, one for shape and one for dimension, suffice for accurate description and determination of area. Indeed, in vegetative leaves it is the area which is main determinant for how much photosynthetic activity can be performed and the contribution of leaves to total biomass of plants. These results led to very simple and accurate methods of area measurement for leaves by demonstrating the large-scale validity of the Montgomery equation, to estimate biomass in leaves. From a geometrical perspective, if one focusses on area instead of form and shape, all leaves can be considered as variations on a single theme: the elliptic leaf, elongated or not. Most recently, the original hypothesis has been validated on 750 sections of the square bamboo Chimonobambusa utilis [3]. A total of 1400 inner and outer shapes were tested. One result was that both super- and subellipses could be distinguished. Another result is that a single deformation parameter suffices to describe deformations of the cross section along the height, due to internal and external forces acting during growth and development, which will allow a better understanding of the relationship between form and function.

REFERENCES

[1] G. Lamé, Examen des différentes méthodes employées pour résoudre les problèmes de géométrie. V. Courcier: Paris, France, 1818.

[2] J. Gielis, The Geometrical Beauty of Plants. Atlantis Press: Paris, France, 2017.

[3] W. Huang, Y. Li, KJ Niklas, J. Gielis, Y. Ding, L. Cao, P. Shi, A superellipse with deformation and its application in describing the cross-sectional shapes of a square bamboo. Symmetry, 2020

Harmonic metrics, harmonic tensors and identity maps

Bang-Yen Chen

Department of Mathematics, Michigan State University, U.S.A.

The notions of harmonic metrics, harmonic tensors were introduced by B.-Y. Chen and T. Nagano in 1984. Since then, harmonic metrics and harmonic tensors have been studied by various authors and many interesting results were obtained.

In this talk I will present a survey on harmonic metrics and harmonic tensors from various geometric and physical points of view.