Technical University of Civil Engineering Bucharest
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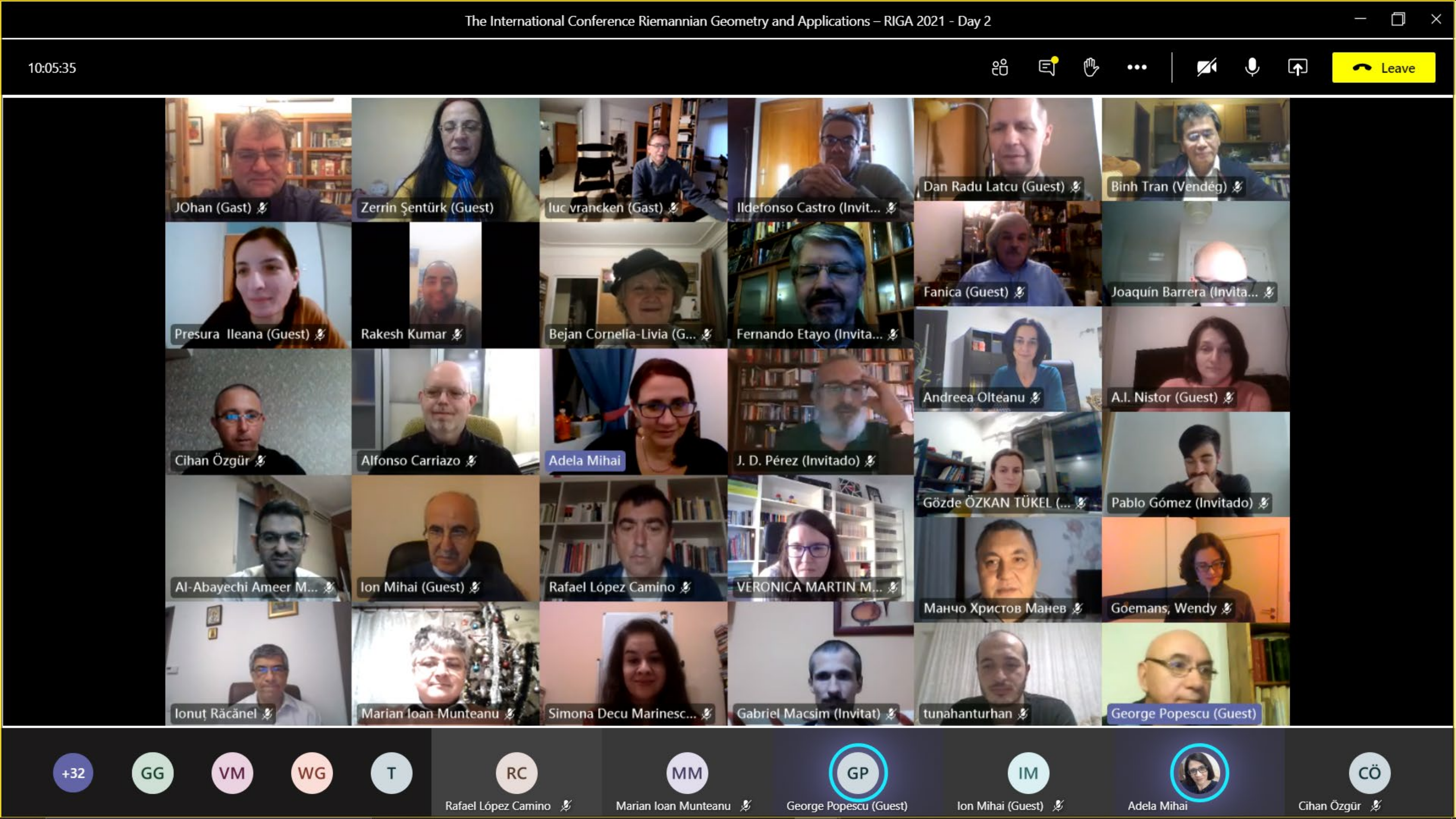
University of Bucharest
Faculty of Mathematics and Computer Science

The International Conference Riemannian Geometry and Applications - Day 2

– RIGA 2021-

Bucharest, Romania, January 16 2021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Statistical structures on Lagrangian submanifolds

$\tilde{M}(4c)$ – complex space form of holomorphic sectional curvature $4c$
M – Lagrangian submanifold,
M – real submanifold of $\tilde{M}$, $\dim M = \dim_{\mathbb{C}} \tilde{M}$, JTM is orthogonal to TM, where J is the complex structure on $\tilde{M}$
g – induced metric tensor field on M
$K = J \circ \sigma$, σ is the second fundamental tensor
K – symmetric and symmetric relative to g
(g, K) – the induced statistical structure on a Lagrangian submanifold
The Codazzi equation for a Lagrangian submanifold says that the $(1,3)$-tensor field $\hat{\nabla} K$ is symmetric. Hence the statistical structure on a Lagrangian submanifold in a complex space form is conjugate symmetric.

which is independent of the choice of the orthonormal basis.

From (2.4), it follows that a Hessian manifold of constant Hessian sectional curvature c is a Riemannian space form of constant sectional curvature $-c$.

Next, a statistical manifold $(\tilde{M}^m, g)$ and a submanifold M^n of dimension n of $\tilde{M}^m$ are considered. Then (M^n, g) is also a statistical manifold with the induced connection by $\tilde{\nabla}$ and induced metric g.

In Riemannian geometry, the fundamental equations are the Gauss and Weingarten formulae and the equations of Gauss, Codazzi and Ricci.

As usual, we denote by $\Gamma\left(T^{\perp}M^n\right)$ the set of the sections of the normal bundle to M^n.

In our case, for any $X, Y \in \Gamma\left(TM^n\right)$, according to [13], the corresponding Gauss formulae are

$$\tilde{\nabla}_X Y = \nabla_X Y + h\left(X, Y\right), \tag{2.5}$$

$$\tilde{\nabla}^*_X Y = \nabla^*_X Y + h^*\left(X, Y\right), \tag{2.6}$$

where $h,\ h^* : \Gamma(TM^n) \times \Gamma(TM^n) \to \Gamma(T^{\perp}M^n)$ are symmetric and bilinear, called the *imbedding curvature tensor* (see [1, 13]) of M^n in $\tilde{M}^m$ for $\tilde{\nabla}$ and the *imbedding curvature tensor* of M^n in $\tilde{M}^m$ for $\tilde{\nabla}^*$

Purposes

- We obtain optimal inequalities involving:
  - the scalar curvature (intrinsic invariant) and
  - the δ-Casorati curvatures (extrinsic invariants)

  of a statistical submanifold in holomorphic statistical manifolds with constant holomorphic sectional curvature.

- We investigate the Casorati ideal submanifolds which characterise the totally geodesic submanifolds with respect to the Levi-Civita connection.

- Example

[S. Decu, S. Haesen and L. Verstraelen, *Inequalities for the Casorati Curvature of Statistical Manifolds in Holomorphic Statistical Manifolds of Constant Holomorphic Curvature*, Mathematics **8 (2)** (2020)]

Simona Decu Marinescu (Guest)

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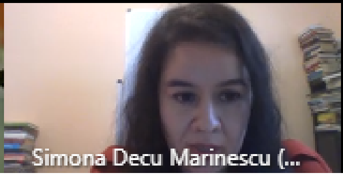


Introduction

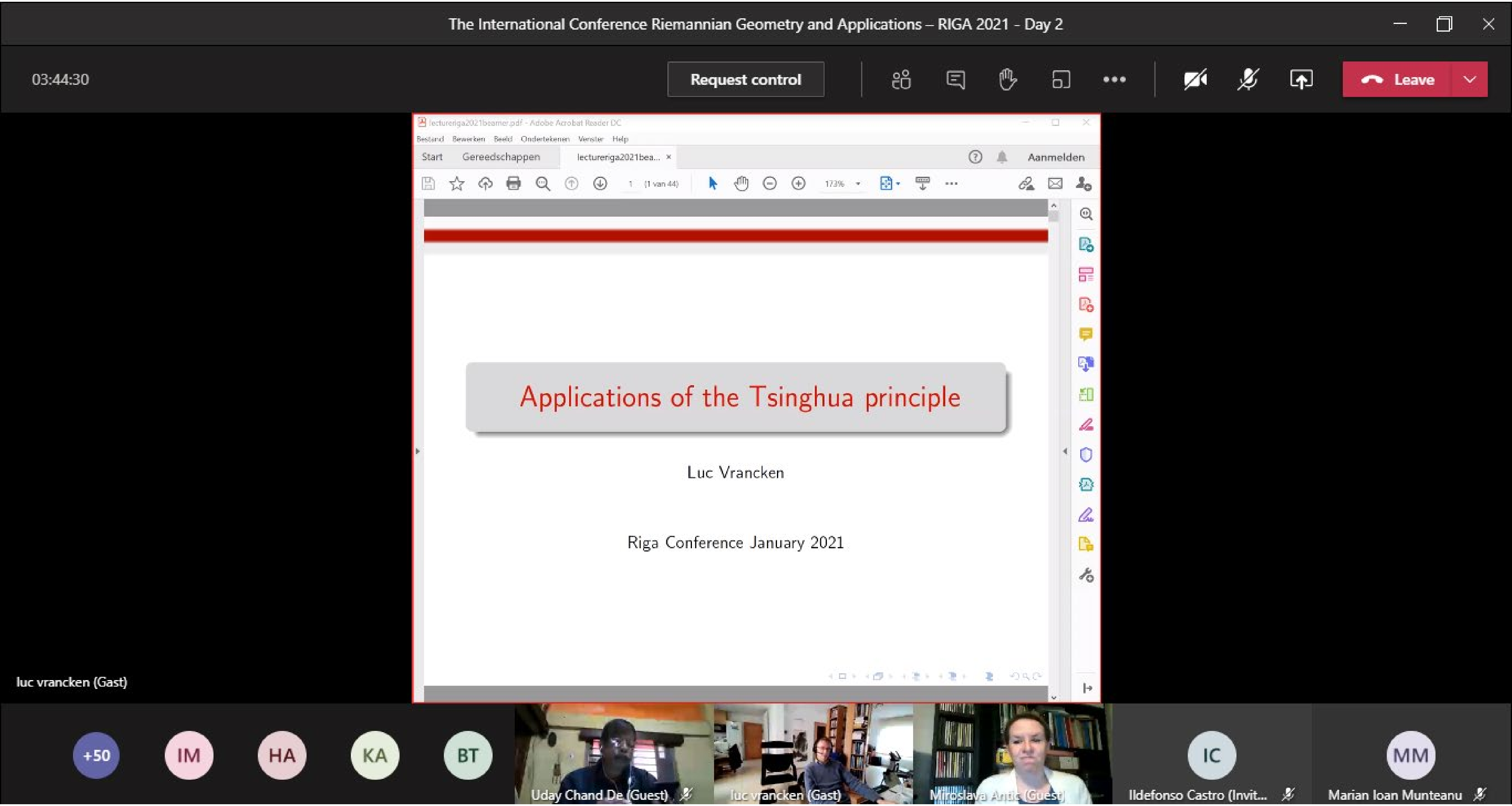
M^n- Blaschke hypersurfaces of the affine space R^{n+1}, D-flat connection.

Formulae of Gauss and Weingarten are
$D_X Y = \nabla_X Y + h(X, Y)\xi, \quad D_X \xi = -SX + \tau(X)$.
∇-induced connection, h-second fundamental form, *S-shape op.*

We assume: *rank* $h = n \Rightarrow M$ is *non-degenerate*

ξ is the Blaschke normal: ω - volume form in R^{n+1}
$\theta(X_1, \dots, X_n) = \omega(X_1, \dots, X_n, \xi) = \det(h(X_i, X_j))$ and its parallel ($\nabla\theta = 0 \Leftrightarrow \tau = 0$-apolarity condition).

Moreover, we assume that h is positive definite, i.e. the hypersurface is locally strongly convex.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Applications of the Tsinghua principle
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In submanifold geometry one studies properties of a submanifold M of N which are invariant under a group of transformations of N.
It originates with the study of curves in $\mathbb{R}^2$ and surfaces in $\mathbb{R}^3$ which are invariant under euclidean transformations (isometries of $\mathbb{R}^n$).
Later one can either look at more general ambient spaces (provided that they have sufficiently many isometries) or look at a more general class of transformations (which is for example done in affine differential geometry)
The ambiant spaces which we will consider in this talk are

- the real space forms
- the complex projective space (or complex space forms)
- the complex quadric
- the complex hyperbolic quadric
- the nearly Kaehler $\mathbb{S}^3 \times \mathbb{S}^3$

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$$R(X,Y)Z = \left(\tilde{R}(X,Y)Z\right)^t + A_{h(Y,Z)}X - A_{h(X,Z)}Y$$

curv. tensor subm — tangential part ambient space

$$\langle R(X,Y)Z,W\rangle = \langle \tilde{R}(X,Y)Z,W\rangle + \langle h(Y,Z),h(X,W)\rangle - \langle h(X,Z),h(Y,W)\rangle$$

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$S^3 \times S^3$

point (p, q)

↳ unit quaternions

$$J_{(p,q)}(pU, qV) = \frac{1}{\sqrt{3}}\left(p(2V-U), q(-2U+V)\right)$$

$J^2 = -I$ not compatible with product metric $S^3 \times S^3$

$$g(Z, Z') = \frac{1}{2}\left(\langle Z, Z'\rangle + \langle JZ, JZ'\rangle\right)$$

luc vrancken (Gast)

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Adara Blaga (Guest)

Ahmet YILDIZ (Konuk)

Alfonso Carriazo — Outside your organization

amrinder pal singh

Andreea Olteanu — Outside your organization

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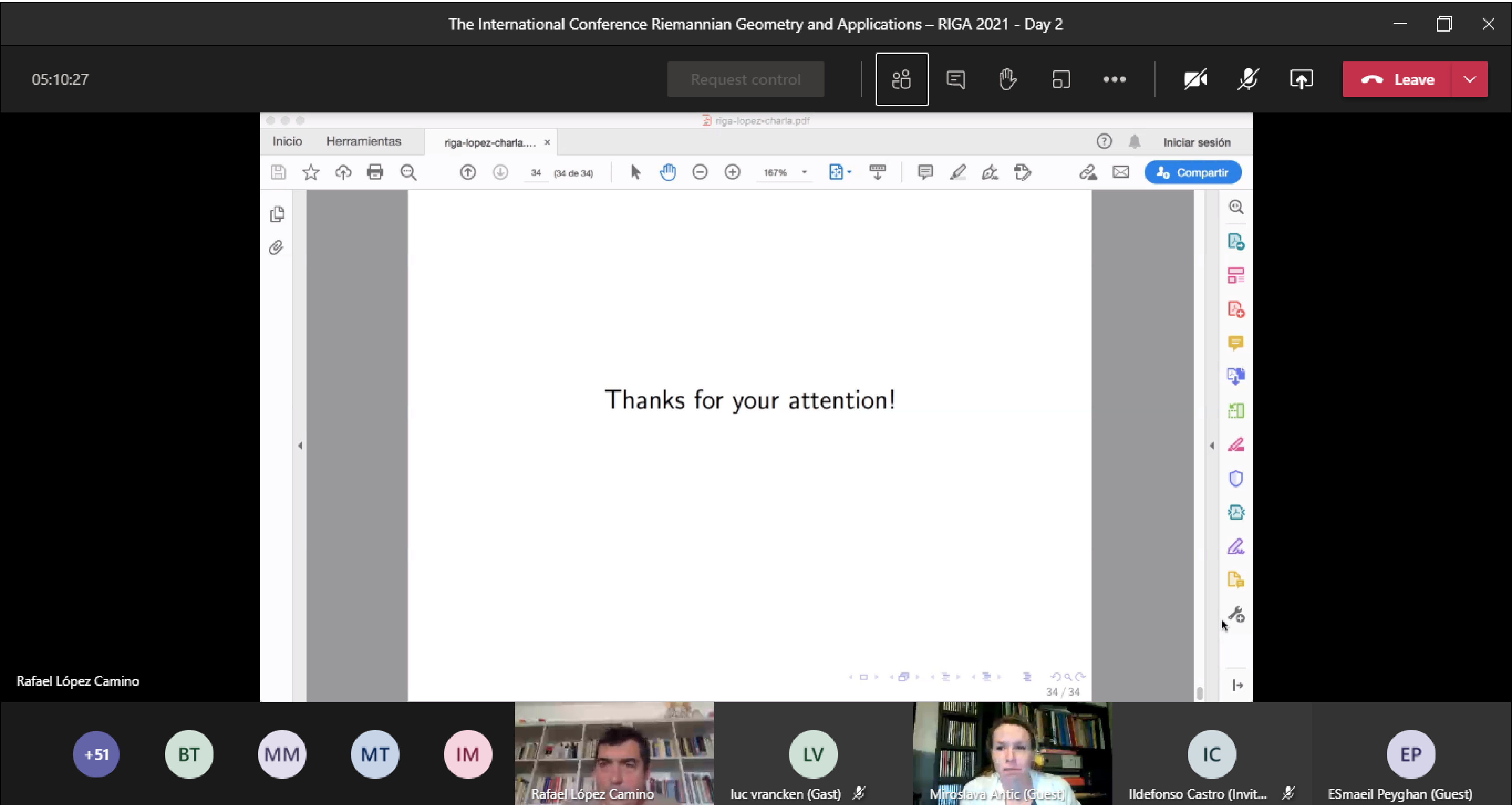
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Then we have

$$(\nabla_Z S)(X,Y) = 2A(Z)S(X,Y) + A(X)S(Y,Z) + A(Y)S(X,Z). \quad (22)$$

Using (21)in (11), we obtain

$$(\nabla_Z T)(X,Y) = 2A(Z)T(X,Y) + A(X)T(Y,Z) + A(Y)T(X,Z) + \frac{1}{\kappa}\{rA(Z)g(X,Y) + \frac{r}{2}A(X)g(Y,Z) + \frac{r}{2}A(Y)g(X,Z) - \frac{1}{2}dr(Z)g(X,Y)\}. \quad (23)$$

In virtue of equations (21) and (23), proof of the Theorem 2 follows.

Constructions and statements of problems

Examples of singular minimal surfaces

Example (continuation)

By multiplying last eq. with $2f'$ and taking first integral, we obtain

$$f' = \sqrt{cf^4 - 1} \text{ and } f'' = 2c_1 f^3, \; c_1 \neq 0.$$

which is known as *Emden–Fowler* equation [PZ] and the solution follows

$$s = \int \left[c_1 f^4 + c_2\right]^{-1/2} df + c_3, \; c_2, c_3 \in \mathbb{R}.$$

Polyanin A. D. Polyanin and V. F. Zaitsev, Handbook of Exact Solutions for Ordinary Differential Equations, Chapman & Hall/CRC, Boca Raton, Fla, USA, 2nd Ed., 2003.

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Slant submanifolds in generalized Sasakian space forms satisfying a natural equality

Pablo Alegre, Joaquín Barrera and Alfonso Carriazo

Department of Geometry and Topology

UNIVERSIDAD DE SEVILLA

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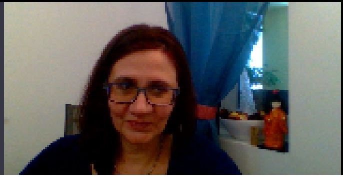


Motivation

Let $M^m \hookrightarrow \mathbb{R}^{2m+1}$ be an **anti-invariant** submanifold.

It was proved by [–, D. E. Blair (2000)] that

$$\|\mathbf{H}\|^2 \geq \frac{2(m+2)}{(m+1)^2(m-1)}\tau,$$

with equality holding if and only if M is either totally contact geodesic or the Riemannian product of a (piece of a) Whitney sphere times $\mathbb{R}$.

Moreover, M satisfies the equality case at every point if and only if its second fundamental form σ is given by

$$\sigma(X,Y) = \frac{m+1}{m+2}\{(g(X,Y) - \eta(X)\eta(Y))\mathbf{H} + (g(\phi X, \mathbf{H}) - \frac{m+2}{m+1}\eta(X))\phi Y + (g(\phi Y, \mathbf{H}) - \frac{m+2}{m+1}\eta(Y))\phi X\}.$$

Notation I

$\mathbb{C}P^m$: complex projective space with complex dimension m equipped with the Kählerian structure (J, g) of constant holomorphic sectional curvature 4.

M: a real hypersurface in $\mathbb{C}P^m$ with unit normal vector field N.

A: shape operator associated to N.

∇: Levi-Civita connection on M.

$\xi = -JN$: Reeb (structure) vector field on M.

$JX = \phi X + \eta(X)N$, ϕX: tangent component of JX, for any $X \in TM$; $\eta(X) = g(X, \xi)$.

Therefore (ϕ, ξ, η, g) defines an almost contact metric structure on M.

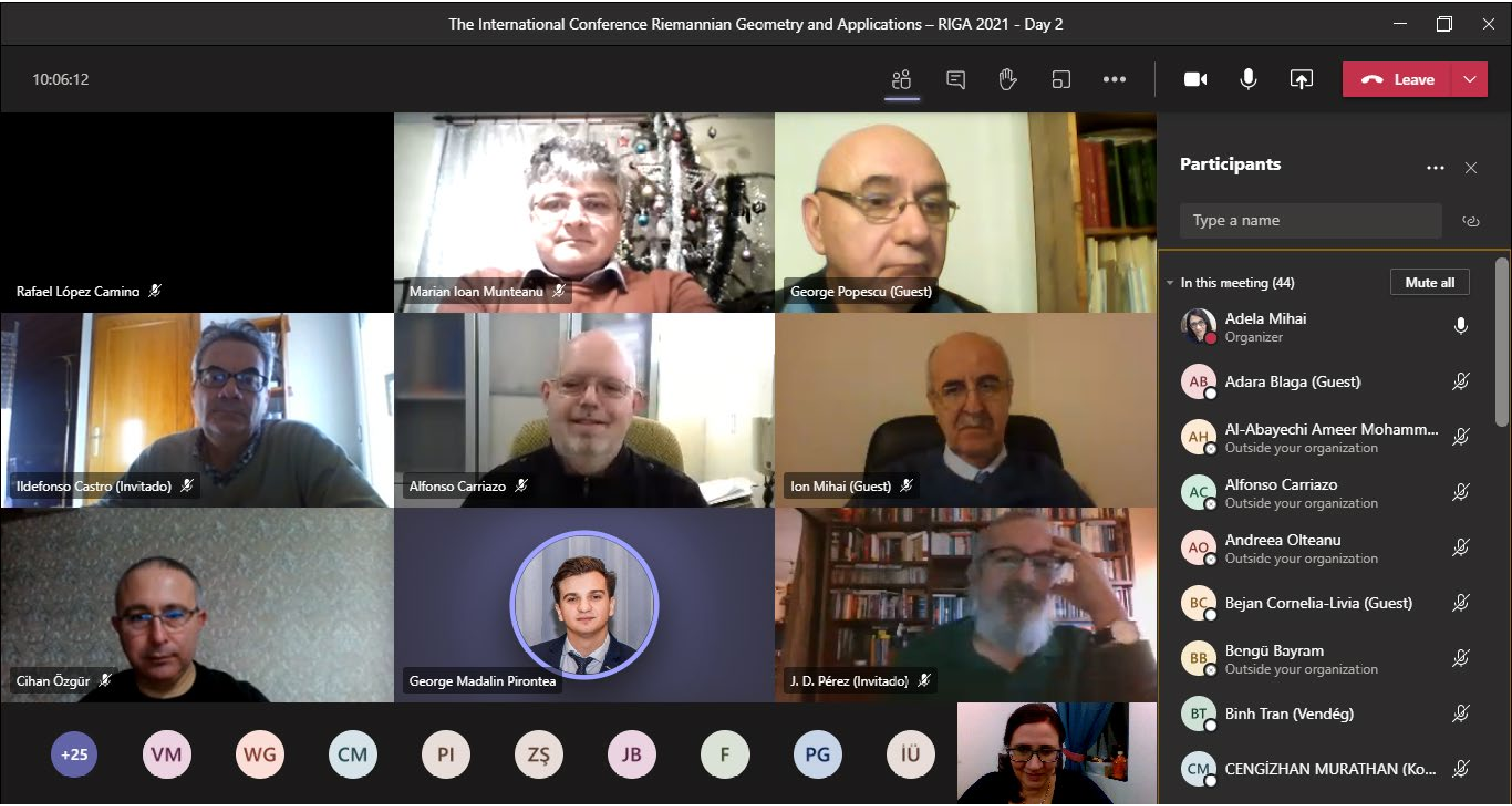
M is called Hopf if ξ is principal, that is, $A\xi = \alpha\xi$.

$\mathbb{D}$: maximal holomorphic distribution on M, given by

$$\mathbb{D}(p) = \{X \in T_pM \,/\, g(X, \xi_p) = 0\}.$$

In [Sular, Özgür, 2012], we studied quasi-Einstein warped product manifolds for arbitrary dimension $n \geq 3$. In [Dumitru, 2012], Dumitru studied the expressions of the Ricci tensors and scalar curvatures for the bases and fibres on quasi-Einstein warped product manifolds. In [Pahan, Pal, 2019], Pahan and Pal studied the Einstein sequential warped product space with negative scalar curvature and gave an example of the Einstein sequential warped space. In [Şahin, 2020], B. Şahin introduced sequential warped product submanifolds of Kaehler manifolds and obtained some examples.

Motivated by the above studies, we consider quasi-Einstein sequential warped product manifolds.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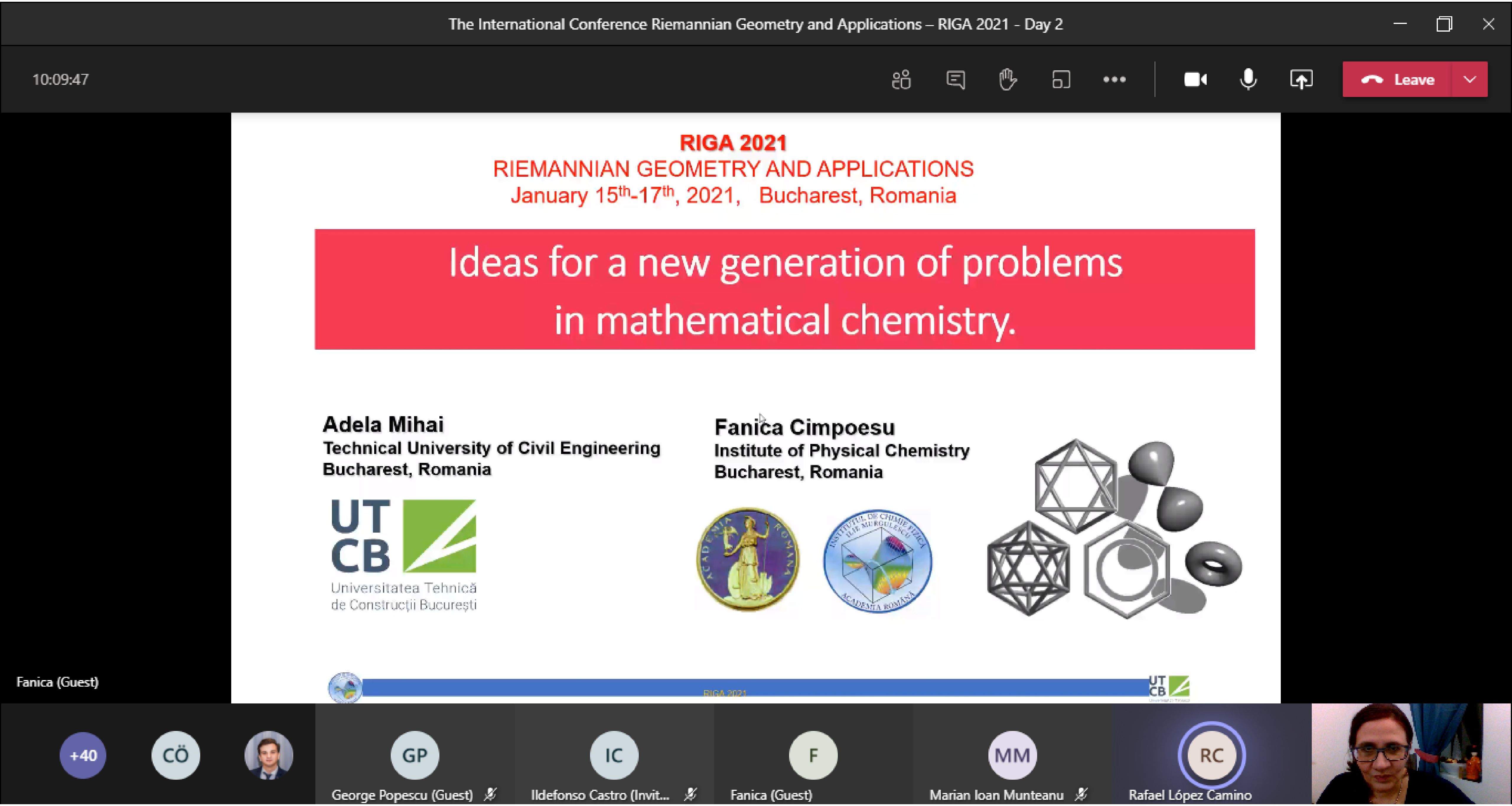
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RIEMANNIAN GEOMETRY AND APPLICATIONS
January 15th-17th, 2021, Bucharest, Romania
Ideas for a new generation of problems in mathematical chemistry.
Adela Mihai
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The practical approximations in quantum mechanics and computational chemistry.

Many solutions to Schrodinger equations. Eigenvalues and eigenvectors.

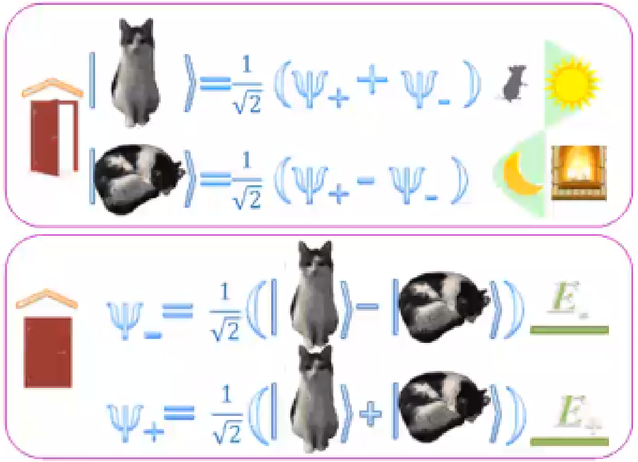
$$\hat{H}\Psi_I = E_I\Psi_I \qquad \int_{\substack{all \\ V\ space}} \Psi_I^*\Psi_J dV = \delta_{IJ}$$

Matrix formulation on quantum mechanics

Werner Heisemberg 1901-1976

Assume a linear decomposition in a basis of a sort of guessed components.
Turn the differential equation in a set of integral equations. Arriving at eigenvectors-eigenvalue problems

$$\Psi_I = \sum_i c_{iI}\varphi_i \qquad \int_{\substack{all \\ V\ space}} \varphi_i^*\hat{H}\varphi_j dV = H_{ij} \qquad \int_{\substack{all \\ V\ space}} \varphi_i^*\varphi_j dV = S_{ij} \qquad \det|H_{ij} - ES_{ij}| = 0$$



Getting rid of quantum dilemmas: "shut up and calculate!"

Cat in a "ket". **The Schrödinger's cat though experiment,** slightly modified: awake vs. asleep cat (not alive vs. dead!). As function of "closed door" vs. "open door" proposed operator (note the hat symbol beneath door's icon), the solution are of "delocalized" vs. localized type. Note that functions with equal probability of standing or sleeping cat are two: ψ_+= in-phase and ψ_-=out-of-phase. This couple has no intuitional meaning, but their remixing (sum and difference, as represented in the top half of the figure) gives rise to "measurable" states of awake or sleeping cat.

Mihai Eminescu
15 Jan. 1850-15 July 1889

Instead of Conclusion

Fanica Cimpoeşu,
Marilena Ferbinteanu,
Adela Mihai,
Ionel Humelnicu

A singular quatrain.
And a whole Universe

Original

Când aduce blonda Liză
Socoteala unei vedre,
Universul cristaliză
Hexacontatetraedre

Thank You!
Multumesc!
La Multi Ani!

The symmetry blueprints of the molecular edifices
Vol. 19, No. 4, 397-414 (2008)

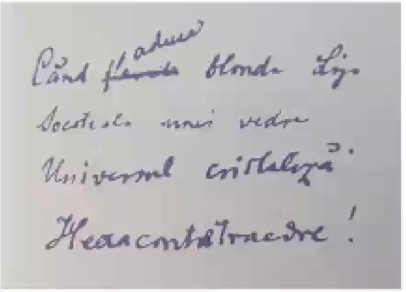
Hexacontatetrahedrom,
the polyhedron with 64 faces,
and the construction stages
(left side panel).

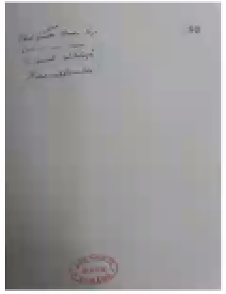
Constrained translation

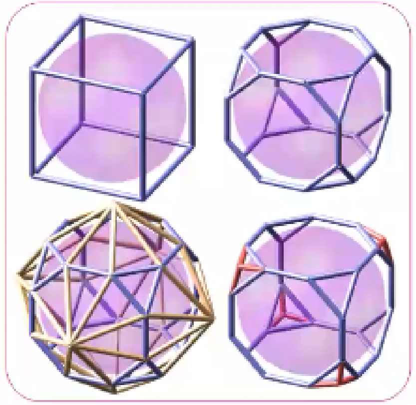
When blonde Lisa brings
the bill of a pint,
the Universe is crystallizing
hexacontetrahedra

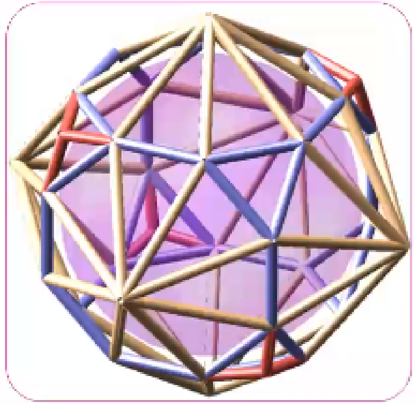
Loose translation

Cute blonde Lisa brings the bill
of my inspiriting beer.
Universe is then a deal
Of supersymmetric stir.









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