

**Technical University of Civil Engineering Bucharest**  
Faculty of Railways, Roads and Bridges

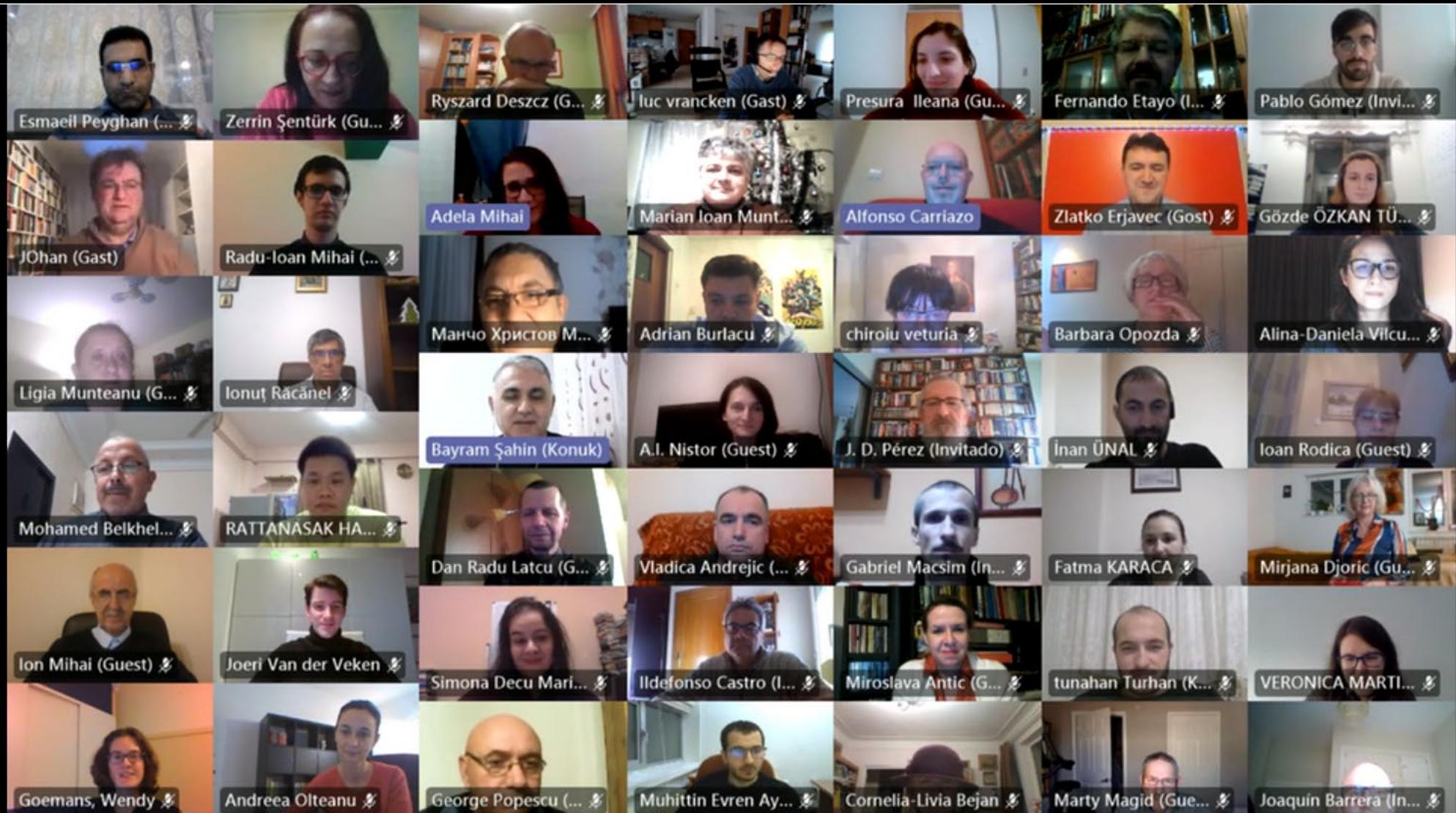
**University of Bucharest**  
Faculty of Mathematics and Computer Science

## **The International Conference Riemannian Geometry and Applications - Day 3**

**- RIGA 2021 -**

**Bucharest, Romania, January 17 2021**

08:24:47



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JOhan (Gast)

Bang-Yen Chen (Guest)

Bayram Şahin (Konuk)

Adela Mihai

Shihshu Walter Wei (Guest)

Marian Ioan Munteanu (Guest)

52:43



On the existence of convex functions on non-compact Finsler manifolds  
Sabau-2020\_RIGA.pdf

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## §1. Preliminaries

**Definition**

A Finsler norm, or metric, on a real smooth,  $n$ -dimensional manifold  $M$  is a function  $F : TM \rightarrow [0, \infty)$  that is positive and smooth on  $\widetilde{TM} = TM \setminus \{0\}$ , has the *homogeneity property*  $F(x, \lambda v) = \lambda F(x, v)$ , for all  $\lambda > 0$  and all  $v \in T_x M$ , having also the *strong convexity* property that the Hessian matrix

$$g_{ij} = \frac{1}{2} \frac{\partial^2 F^2}{\partial y^i \partial y^j}$$

is positive definite at any point  $u = (x^i, y^i) \in \widetilde{TM}$ .  
 $(M, F)$  ia called a *Finsler manifold* or *Finsler structure*.

Sorin V. Sabau 2 / 47

**Participants**

Type a name

Name	Type	
Joaquin Barrera (Invitado)		
JOhan (Gast)		
Ligia Munteanu (Guest)		
Miroslava Antic (Guest)		
Muhittin Evren Aydin (Konuk)		
Mukut Mani Tripathi (Guest)		
Nico Voicu (Invitat)		
Pablo Alegre (Invitado)		
Pablo Gómez (Invitado)		
Rafael López Camino Outside your organization		

Sorin V. SABAU

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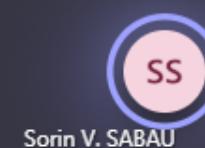
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RAKESH KUMAR (Gue...



02:51:20

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10 (10 of 10) 154% Tools Fill & Sign Comment

## Some Valuable References

- C. Udriște, *Convex functions and optimization methods in Riemannian manifolds*, Mathematics and Its Applications, Vol. 297, Kluwer Academic, Dordrecht, 1994.
- M. M. Postnikov, *Geometry VI: Riemannian geometry*, Springer-Verlag, 2001.

Esa Sharahi (Guest)

Finding Geodesics on Surfaces Using Taylor Ex

**Participants**

Type a name

Outside your organization

Ildefonso Castro (Invitado)

 İnan ÜNAL  
Outside your organization

Ioan Rodica (Guest)

 ION MIHAI  
Outside your organization AC The International Conference...  
Very nice talk. Congratulations!

Reply

 MT The International Conference...  
Nice tlk. Thank you Prof Sharahi

Reply

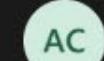
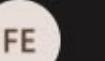
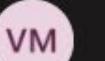
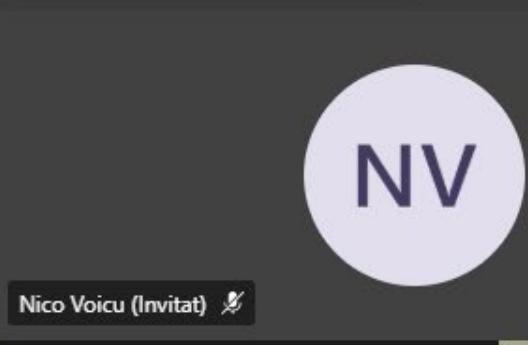
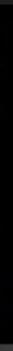
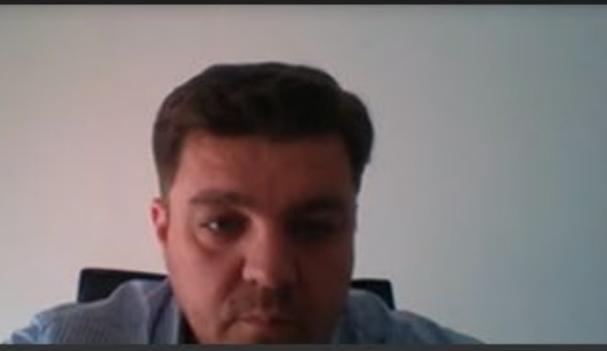


Esa Sharahi (Guest)

Nico Voicu (Invitat)

Gabriel Macsim (Invitat)

03:28:35



03:56:51

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RIGA 2021 prezentare.pptx [Vizualizare protejată] - PowerPoint

Conectare

Fisier Pornire Inserare Proiectare Tranzitii Animații Expunere diapoziție Revizuire Vizualizare MathType Ajutor ACROBAT Spune-mi ce dorești să fac

Consider next, the Liouville–Tzitzieica equation

$u_x = \exp u , \quad (5)$

$v_x = 0 . \quad (6)$

$u_x + v_x = \sqrt{2} \exp \frac{u-v}{2} , \quad u_x - v_x = \sqrt{2} \exp \frac{u+v}{2} . \quad (7)$

$u_x + v_x = \frac{1}{\sqrt{2}}(u_x - v_x) \exp \frac{u-v}{2} = \exp u , \quad (8)$

$u_x - v_x = \frac{1}{\sqrt{2}}(u_x + v_x) \exp \frac{u+v}{2} = \exp u ,$

27

Diapoziție 27 din 64 Engleză (Statele Unite)

Comentarii

80%

Ligia Munteanu (Guest)

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Koji Matsumoto (ゲスト)

IR



Ioan Rodica (Guest)

Adrian Burlacu



Ligia Munteanu (Guest)

06:15:27



1. Introduction
2. The invariant  $\delta_k$
3. An inequality for the invariant  $\delta_k$
4. The equality case

## Basic notions

## Examples

The global closed 1-form  $\alpha$ *Example 2.* Diagonal Hopf manifolds.<sup>5</sup>

One considers  $H_A = (\mathbb{C}^n)/\langle A \rangle$  with  $A = \text{diag}(\alpha_i)$  endowed with

- Complex structure as the projection of the standard one of  $\mathbb{C}^n$ .
- I.c.K. structure constructed as follows:

Let  $C > 1$  be a constant and

$$\varphi(z_1, \dots, z_n) = \sum |z_i|^{\beta_i}, \quad \beta_i = \log_{|\alpha_i|^{-1}} C$$

a potential on  $\mathbb{C}^n$ .

Then  $A^* \varphi = C^{-1} \varphi$ . Thus  $\Omega = \sqrt{-1} \partial \bar{\partial}$  is Kähler.

The Lee field  $\theta^\# = - \sum z_i \log |\alpha_i| \partial z_i$  is parallel.

---

<sup>5</sup>I. Verbitsky, *Theorems on the vanishing of cohomology for locally conformally hyper-Kähler manifolds*, (Russian) Tr. Mat. Inst. Steklova **246** (2004), Algebr. Geom. Metody, Svyazi i Prilozh., 64–91; translation in Proc. Steklov Inst. Math. 246 (2004), 54–78.

RIGA, January 15-17, 2021

Invariants on locally conformal Kähler manifolds

Gabriel Macsim (Invitat)

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Mirjana Djoric (Guest)

Cihan Özgür

Gabriel Macsim (Invitat)

Type a new message



## Meeting chat

Zlatko Erjavec (Guest) (Guest)  
3:42 PM  
Hvala na krasnom  
izlaganju profesorice Đorić!

Doç.Dr. Mehmet Akif AKYOL  
joined the meeting.  
Vladica Andrejic (Guest) left  
the meeting.

Siraj Uddin (Jeddah) (Guest)  
3:44 PM  
Thank you, Professor Mirjana,  
Very nice talk and a lot of  
information

nice talk

Miroslava Antic (Guest) left the  
meeting.

06:38:49

**Request control****Leave**

Duality in Differential Equations and Riemann – Finsler Geometry Xinjiang University 071517 copy.pdf

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## Prove Dualities in Differential Eq. and Dif. inequalities

( $\Rightarrow$ ) Now we set  $g = \frac{\kappa f'}{f}$ . and  $g$  is well-defined function in  $(0, \tau)$ .

It follows from the quotient law, (1), and (4) that

$$\begin{aligned} g' &= \frac{\kappa f''f - \kappa(f')^2}{f^2} \\ &= (\text{resp. } \leq, \geq) \frac{-\kappa Gf^2}{f^2} - \frac{\left(\frac{\kappa f'}{f}\right)^2}{\kappa} \\ &= (\text{resp. } \leq, \geq) -\kappa G - \frac{g^2}{\kappa} \quad \text{a. e. in } (0, \tau). \end{aligned}$$

Thus we obtain a first-order nonlinear differential equation

$$g' + \frac{g^2}{\kappa} + \kappa G = 0 \quad (\text{resp. } \leq 0, \geq 0)$$

with the initial conditions at 0 converted to the asymptotic one at 0 :

$$g(t) = \kappa \frac{f'(t)}{f(t)} = \kappa \frac{\kappa + O(t)}{\kappa t + O(t^2)} = \frac{\kappa}{t} \cdot \frac{1 + O(t)}{1 + O(t)}$$
**Meeting chat**

Thank you!

4:04 PM  
is just a start , once we will find  
good examples will be more  
meaningfull

Cihan Özgür 4:05 PM  
Sorry Gabriel because of time.  
Congratulations for your  
interesting talk!

Gabriel Macsim (Invitat) (Guest)  
4:05 PM  
It's not a problem. Thank you a  
lot!

tunahan Turhan (Konuk) joined  
the meeting.

Type a new message



Shihshu Walter Wei (Guest)

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Gabriel Macsim (Invitat)

Cihan Özgür

Shihshu Walter Wei (Guest)

07:06:38



Johan (Gast)



Shihshu Walter Wei (Guest)



Siraj Uddin (Jeddah) (Guest)



Marian Ioan Munteanu



Gabriel Macsim (Invitat)



Cihan Özgür



ION MIHAILESCU



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07:07:35

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Lamé curves  
 $(x^n + y^n = R^n)$   
in nature

Johan Gielis

Riemannian Geometry and Applications RIGA 2021

The slide is part of a presentation titled "Riemannian Geometry and Applications RIGA 2021". The background of the slide shows a list of 14 slides, with the current slide being number 11. The right side of the slide has a "Layout dia" ribbon tab open, showing options for "Titel en opsomming" and "Kies basisslides". It also includes sections for "Weergave" (Title, Main title, Document number) and "Achtergrond" (Background color).

Johan (Gast)

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Shihshu Walter Wei (G...)

Cihan Özgür

Radu Ioan Mihai (Gue...)



07:09:11

**Request control****Leave**

Lamé curves  
 $(x^n + y^n = R^n)$   
in nature

Johan Gielis

Riemannian Geometry and Applications RIGA 2021

The presentation slide is part of a deck numbered 1 to 14. The right side of the screen shows the Powerpoint ribbon and a 'Layout dia' panel.

Johan (Gast)

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Shihshu Walter Wei (G...)

Cihan Özgür

Radu Ioan Mihai (Gue...)



06:54:57

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# Comparison Principle, Duality, and Harmonicity - Comparing Diverse Phenomena and Discovering Secret Unity

Shihshu Walter Wei

The University of Oklahoma

Riemannian Geometry and Applications - RIGA 2021  
Bucharest, Romania

Shihshu Walter Wei (Guest)

06:55:45

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Motivation  
Introduction  
Geometrical Approach  
New Ideas

## A Geometric Interpretation of Some Relations Between Integer Sequences

Radu-Ioan MIHAI

Student, Faculty of Mathematics and Computer Science

University of Bucharest, Romania



07:18:57

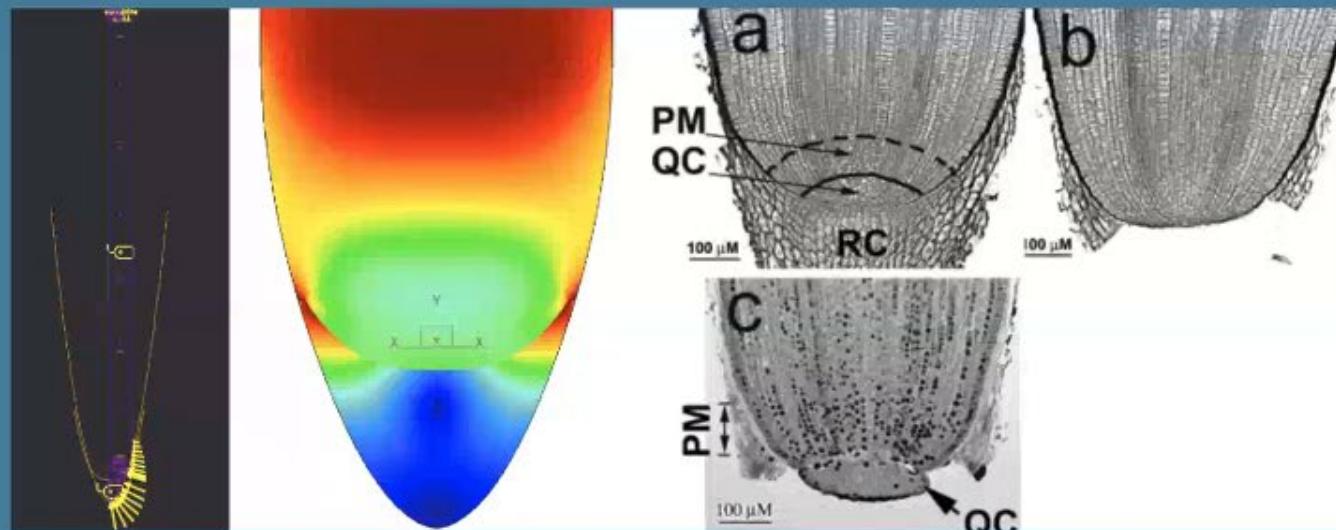
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## Stress and Strain in roots: a natural metamaterial



The structure of root and root cap, create a quiescent centre at the  
**Flackpunkt** of the superellipsoids (or superparaboloid)

Johan (Gast)

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Cihan Özgür

Radu-loan Mihai (Guest)

George Popescu (Guest)



07:40:57



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A presentation slide titled "Harmonic metrics, harmonic tensors and identity map" by Bang-Yen Chen from Michigan State University, presented at the International Conference Riemannian Geometry and Applications – RIGA 2021 on January 17, 2021, in Romania.

Harmonic metrics, harmonic tensors  
and identity map

Bang-Yen Chen

Michigan State University

The International Conference Riemannian Geometry and Applications  
RIGA 2021  
January 17, 2021  
Romania

Bang-Yen Chen (Guest)

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Radu-Ioan Mihai (Gue...)

George Popescu (Guest)



07:52:14



Leave

Two fundamental theorems for harmonic metrics

- Let  $\mathcal{F}(M)$  = the space of smooth functions on  $M$ . The next two theorems describe the space of all harmonic tensors of  $(M, g)$ .

**Theorem 3.** If  $(M, g)$  is a Riemannian manifold with  $\dim M \geq 3$ , then we have the following linear isomorphism:

$$\{\text{Harmonic metrics of } M\} \cong \{T \in \mathcal{S} : \delta T = 0\} = \{\text{Conservative metrics of } M\}.$$

This isomorphism is given by  $\mathcal{S} \ni g' \mapsto T_{g'} = g' - \frac{1}{2}(\text{tr}_g g')g \in \ker(\delta)$ .

- Theorem 4.** If  $\dim M = 2$ , then

$$\{\text{Harmonic metrics of } (M, g)\} = \{\text{Conservative metrics of } (M, g)\} \oplus \{\lambda g : \lambda \in \mathcal{F}(M)\},$$

where  $\{\lambda g : \lambda \in \mathcal{F}(M)\}$  is the set of all conformal changes of metric  $g$ .

Bang-Yen Chen (Guest)

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Radu-loan Mihai (Gue...)

George Popescu (Guest)



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Bang-Yen Chen (Guest)

08:00:55



**Tadashi Nagano, Kentaro Yano and Katsumi Nomizu**



Yano and Nomizu.jpg

Tadashi Nagano (1930–2017) (left),  
Kentaro Yano (1912–1993) (center),  
Katsumi Nomizu (1924–2008) (right).

## Participants

Type a name

In this meeting (78)

Mute all

-  Adela Mihai  
Organizer
-  A.I. Nistor (Guest)
-  Adara Blaga (Guest)
-  Ahmet YILDIZ (Konuk)
-  Al-Abayechi Ameer Mohamm...  
Outside your organization
-  alegra
-  Alfonso Carriazo  
Outside your organization
-  Alina-Daniela Vilcu (Invitat)
-  Andreea Olteanu  
Outside your organization

Bang-Yen Chen (Guest)

+69

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Bayram Şahin (Konuk)

J

BC

Johan (Gast)

Bang-Yen Chen (Guest)

08:01:02



Leave

Kentaro Yano at Michigan State University

and Chen.jpg

Yano and Chen at East Lansing, Michigan on May 14, 1972.

Participants

Type a name

In this meeting (78)

Mute all

Adela Mihai  
Organizer

A.I. Nistor (Guest)

Adara Blaga (Guest)

Ahmet YILDIZ (Konuk)

Al-Abayechi Ameer Mohamm...  
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alegra

Alfonso Carriazo  
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Alina-Daniela Vilcu (Invitat)

Andreea Olteanu  
Outside your organization

Bang-Yen Chen (Guest)

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Bayram Şahin (Konuk)

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BC

Johan (Gast)

Bang-Yen Chen (Guest)

08:01:11



Leave

Nagano and Chen in Tokyo, Japan

T. Nagano, B. Y. Chen, Y. Ohnita and Y. Tazawa in Tokyo 1998.

Bang-Yen Chen (Guest)

+69

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GP



Bayram Şahin (Konuk)

J



Bang-Yen Chen (Guest)

## Participants

Type a name

In this meeting (78)

Mute all

Adela Mihai  
Organizer

A.I. Nistor (Guest)

Adara Blaga (Guest)

Ahmet YILDIZ (Konuk)

Al-Abayechi Ameer Mohamm...  
Outside your organization

alegra

Alfonso Carriazo  
Outside your organization

Alina-Daniela Vilcu (Invitat)

Andreea Olteanu  
Outside your organization

08:25:00



Change scene ↎

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Adela Mihai

Alfonso Carriazo

Bang-Yen Chen (Guest)

Bayram Şahin (Konuk)

Shihshu Walter Wei (Guest)

Marian Ioan Munteanu