RIGA 2023

September 22-24, 2023

Bucharest, Romania

BOOK OF ABSTRACTS

Riemannian Geometry and Applications - RIGA 2023
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SUB-FINSLER GEOMETRY AND NONHOLONOMIC MECHANICS

LAYTH M. ALABDULSADA

Abstract. In this talk, we introduce the notion of the nonholonomic sub-Finslerian structure and prove that the distributions are geodesically invariant concerning the Barthel non-linear connection. We provide necessary and sufficient conditions for the existence of the curves that are abnormal extremals; likewise, we provide necessary and sufficient conditions for normal extremals to be the motion of a free nonholonomic mechanical system, and vice versa. Moreover, we show that a coordinate-free approach for a free particle is a comparison between the solutions of the nonholonomic mechanical problem and the solutions of the Vakonomic dynamical problem for the nonholonomic sub-Finslerian structure. In addition, we provide an example of the nonholonomic sub-Finslerian structure. Finally, we show that the sub-Laplacian measures the curvature of the nonholonomic sub-Finslerian structure.

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TOTALLY GEODESIC SUBALGEBRAS OF 6-DIMENSIONAL NILMANIFOLDS HAVING NILPOTENCY CLASSES 3 AND 4

SAMEER ANNON ABBAS[1], ÁGOTA FIGULA[2]

Abstract. We call the metric Lie algebra n having a framing determined by ideals if it has a descending series of ideals invariant under all automorphisms of n and the dimension of the consecutive members of the series decreases by one. We study the sets of geodesic vectors and flat totally geodesic subalgebras in this class of metric Lie algebras. Then determine the geodesic vectors and flat totally geodesic subalgebras in metric Lie algebras of dimension 6 having nilpotency class 3 and 4.

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A CLASS OF CONFORMALLY FLAT AFFINE HYPERSURFACES

MIROSLAVA ANTIĆ[1], HAIZHONG LI[2], LUC VRANCKEN[3], XIANFENG WANG[4]

Abstract. We investigate conformally flat, affine hypersurfaces in terms of multiplicity of the eigenvalues of its Shouten tensor and give the full classification of those admitting at most one eigenvalue of multiplicity one.

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ROTATIONALLY SYMMETRIC TRANSLATORS TO GAUSS CURVATURE FLOW

MUHITTIN EVREN AYDIN

Abstract. This talk is based on the two papers [3, 4]. In the Euclidean ambient space, a surface is called $K^\alpha-$translator of the flow by the powers of the Gauss curvature, if satisfies $K^\alpha = \langle N, \vec{v}\rangle$, $\alpha \neq 0$, where K is the Gaussian curvature, N the Gauss map and $\vec{v}$ is a direction. We obtain a result describing all $K^\alpha-$translators that are rotationally symmetric. The result is also extended to the Lorentzian setting.

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SOME REMARKS ON CONTACT-COMPLEX RIEMANNIAN SUBMERSIONS

CORNELIA-LIVIA BEJAN[1], ŞEMSİ EKEN MERİÇ[2], EROL KILIÇ[3]

Abstract. In this paper, we deal with a contact-complex submersion from an almost contact metric manifold $\tilde{M}$ onto an almost Hermitian manifold M. We study here the total manifold $\tilde{M}$ admits an $\eta-$Ricci soliton and using the Ricci tensor of $\tilde{M}$, we obtain some results for constant-complex Riemannian submersions.

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SUBMANIFOLDS AS HYPERBOLIC RICCI SOLITONS

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Abstract. We point out some properties of hyperbolic Ricci solitons with torse-forming, in particular, with torqued potential vector field, providing also conditions for the vector field to be parallel and the soliton to be just an Einstein manifold. We consider Riemannian manifolds endowed with a concurrent vector field and study their submanifolds which are hyperbolic Ricci solitons having as potential vector field the tangential component of it. In particular, we prove that a totally umbilical hyperbolic Ricci soliton is an Einstein manifold. Finally, we show that if the ambient manifold is of constant sectional curvature, and if a hyperbolic Ricci soliton hypersurface has parallel shape operator, then it is a pseudosymmetric and a metallic shaped hypersurface. Furthermore, we provide certain conditions for it to be minimal.

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A GEOMETRIC CHARACTERIZATION OF THE QUADRIC SURFACES OF REVOLUTION

PAULA CARRETERO[1], ILDEFONSO CASTRO [1,2]

Abstract. It is known (see [1]) that the ellipsoid of revolution

$$\frac{x^2+y^2}{a^2}+\frac{z^2}{b^2}=1 \tag{1}$$

satisfies the relation

$$k_{\mathrm{m}}=\frac{a^4}{b^2}\,k_{\mathrm{p}}^3. \tag{2}$$

Here we denote the principal curvatures κ_1 and κ_2 of a rotational surface, whose curvature lines are the meridians (m) and the parallels (p), by k_{m} and k_{p} respectively.

In [3], it is also proved that any *closed* surface of revolution satisfying $k_{\mathrm{m}}=c\,k_{\mathrm{p}}^3$, for any *positive* constant $c>0$, is congruent to some ellipsoid of revolution. The aim of this talk is to generalize this result using our local approach to the study of rotational Weingarten surfaces given in [2] in order to characterize the non-degenerated quadric surfaces of revolution in terms of a cubic Weingarten relation.

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ON THE PATTERN RECOGNITION OF ROBOTIC JOINTS QUALITY

VETURIA CHIROIU, MARIUS IONESCU, LIGIA MUNTEANU, NICOLETA STAN

Abstract. We report in this paper an analysis of an important task in robotics. i.e. to find out which tasks would be reasonable to characterize the bond quality of robotic joints. Detecting stairs and holes for example, depend on the bond quality. If the robot falls downstairs, for example, it not only gets damaged but it could also hurt people. The first step appears as a wall and the detecting is more difficult. The idea of the paper is to show how the transmitted and re?ected ultrasonic waves can solve the dead zone problem where the robot does not recognize the collisions with outlying objects. The numerical simulation of the ultrasonic wave motion through robotic elements uses the local interaction simulation approach (LISA) which is the simplest mode to avoid the dangerous situations. The problem considered in this paper may be very useful to test models for the simulation of ultrasonic wave propagation in different robotic joints.
Keywords: Robotics; Higher harmonics; Nonlinear ultrasound; Numerical simulations

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ON INEQUALITIES FOR GENERALIZED NORMALIZED δ-CASORATI CURVATURES OF SUBMANIFOLDS IN GOLDEN RIEMANNIAN SPACE FORMS

MAJID ALI CHOUDHARY

Abstract. The idea of polynomial structures on a manifold is credited to Goldberg [4, 5], and the structure of the golden type was developed in [1]. On the other hand, The concept of δ-invariants [2] is an interesting area of modern differential geometry with several applications. Decu et al. [3] started the investigation of δ-Casorati curvatures in the spirit of δ-invariants. This article offers a thorough analysis of current advancements for δ-Casorati curvatures in golden Riemannian space forms made during the previous years.

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THE GEOMETRY OF SEMI-SLANT SUBMERSIONS WHOSE TOTAL MANIFOLDS ARE LOCALLY CONFORMAL KAEHLER MANIFOLDS

ÇAGRIHAN ÇIMEN[1,2], HAKAN METE TAŞTAN[3]

Abstract. We study semi-slant submersions from locally conformal Kaehler manifolds. We first give conditions for distributions which are involved in the definition of the submersion to be integrable. We investigate the harmonicity of such submersions. Moreover, we give a necessary and sufficient condition for these types of submersions to be Clairaut.

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ISOTROPIC SUBMANIFOLDS OF PSEUDO-RIEMANNIAN SPACE FORMS

ALEXANDRU CIOBANU

Abstract. Spacelike and timelike isotropic submanifolds of pseudo-Riemannian spaces have interesting properties, with important applications in Mathematics and Physics. The talk presents inequalities for isotropic spacelike and timelike submanifolds of pseudo-Riemannian space forms. Isotropic Lorentzian submanifolds are also considered.

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IMPACT OF GRADIENT ρ-EINSTEIN SOLITONS IN PERFECT FLUID SPACETIMES

KRISHNENDU DE

Abstract. The present talk concerns with the study of perfect fluid spacetimes equipped with gradient ρ-Einstein solitons. At first, we present the effect of a perfect fluid spacetime with Killing velocity vector admitting a ρ-Einstein soliton of gradient type. Besides, we establish that in a perfect fluid spacetime with constant scalar curvature, if the Lorentzian metric is the gradient ρ-Einstein soliton, then either the ρ-Einstein gradient potential function is pointwise collinear with the velocity vector field, or the spacetime represents stiff matter fluid. Furthermore, we prove that under certain conditions, a perfect fluid spacetime turns into a generalized Robertson-Walker spacetime, as well as a static spacetime.

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CURVATURE INEQUALITIES FOR STATISTICAL SUBMANIFOLDS

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Abstract. A fundamental problem in submanifold theory is to establish optimal relationships between intrinsic and extrinsic invariants of submanifolds. In this respect, B.-Y. Chen introduced in the 1990s new types of intrinsic invariants called d-invariants (now called Chen invariants). Moreover, B.-Y. Chen obtained geometric (Chen) inequalities in terms of these invariants and the extrinsic squared mean curvature, providing solutions of above problem. This approach initiated a new line of research and was developed by many authors. On the other hand, later new types of extrinsic d-Casorati invariants were introduces in [2], involved in optimal (Casorati) inequalities in relation to the intrinsic scalar curvature. There is a great interest also to study Chen inequalities and Casorati inequalities on statistical (sub)manifolds, a concept introduced by Amari in 1985 in the context of information geometry. In this talk, we investigate Chen inequalities and Casorati inequalities on statistical submanifolds in some geometric contexts [1], [2]. Furthermore, we study the equality cases of these inequalities and consider an example.

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COMPACT WEYL-PARALLEL MANIFOLDS

ANDRZEJ DERDZINSKI (JOINT WORK WITH IVO TEREK)

Abstract. ECS manifolds are pseudo-Riemannian manifolds of dimensions $n \geq 4$ which have parallel Weyl tensor, but not for one of two obvious reasons: conformal flatness or local symmetry. They exist for every $n \geq 4$, their metrics are always indefinite, and their local structure has been completely described.

Every ECS manifold has an invariant called rank, equal to 1 or 2. Known examples of compact ECS manifolds, representing every dimension $n > 4$, are all of rank 1. When n is odd, some of the known examples are locally homogeneous.

We prove that a compact rank-one ECS manifold, if not locally homogeneous, replaced if necessary by a two-fold isometric covering, must be the total space of a bundle over the circle.

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HYPERSURFACES IN SPACES OF CONSTANT CURVATURE SATISFYING A PARTICULAR ROTER TYPE EQUATION

RYSZARD DESZCZ, MAŁGORZATA GŁOGOWSKA

Abstract. Let M be a hypersurface in an $(n+1)$-dimensional space of constant curvature N, $n \geq 4$. Let g, R, S, S^2 and $\mathcal{S}$ be the metric tensor, the Riemann-Christoffel curvature tensor, the Ricci tensor and its square, and the Ricci operator of M, respectively. In this talk we present results on hypersurfaces M in N having three distinct principal curvatures obtained in [1]. Among other things we state that on the set U of all points of M at which $\mathcal{S}$ has three distinct Ricci principal curvatures, the tensor R is a linear combination of some Kulkarni-Nomizu products formed by the tensors g, S and S^2, i.e., R satisfies on U a particular Roter type equation. Moreover, the $(0,4)$-tensor $R \cdot S$ is on U a linear combination of some $(0,4)$-tensors formed by g, S and S^2. We refer to [2] and [3] for a survey on manifolds (hypersurfaces) satisfying Roter type equations.

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CLASSIFICATION OF 5-DIMENSIONAL ANTI-COMMUTATIVE LIE ALGEBRAS

KORNÉLIA FICZERE[1], ÁGOTA FIGULA[2]

Abstract. We study 5-dimensional anti-commutative algebras, called $\mathcal{M}^5$-algebras, having the same ideal structure as the 5-dimensional solvable Malcev algebras. These anti-commutative nearly-Malcev algebras are semidirect sums of a 2-dimensional non-abelian and a 3-dimensional abelian Lie algebra. In this presentation I would like to show how we can classify those 5-dimensional anti-commutative algebras which have the same ideal structure as the 5-dimensional solvable Malcev algebras.

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EXTENSIONS AND TANGENT PROLONGATIONS OF DIFFERENTIABLE LOOPS

ÁGOTA FIGULA

Abstract. The tangent Akivis algebra and Sabinin algebra of degree 3 of a differentiable loop is the tangential object determined by the third-order Taylor polynomial of the multiplication function of the loop ([1], [2], [4], [5]). It is endowed with a bilinear skew-symmetric and a trilinear operation defined by the infinitesimal commutator and associator of the loop ([3]). The aim of this talk is to study tangent algebras of degree 3 of abelian extensions of differentiable loops, which are affine extensions of the tangent algebras of the loop by abelian algebras. We apply the obtained results to the determination of tangent algebras of degree 3 of tangent prolongation of differentiable loops.

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EXPOSITION OF CONFORMAL SLANT SUBMERSIONS FROM NEARLY KAEHLER MANIFOLD

GARIMA GUPTA

Abstract. We study slant submersions and conformal slant submersions from nearly Kaehler manifolds onto Riemannian manifolds and investigate conditions for such maps to be totally geodesic maps. We also obtain conditions for a slant submersion and a conformal slant submersion from a nearly Kaehler manifold onto a Riemannian manifold to be a harmonic map and a harmonic morphism, respectively.

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GENERALIZED MÖBIUS-LISTING SURFACES AND BODIES AND APPLICATIONS

JOHAN GIELIS

Abstract. Generalized Möbius-Listing surfaces and bodies are generalizations of the classic Möbius band. In general, cutting leads to interlinked and intertwined different surfaces or bodies, resulting in very complex systems. However, under certain conditions, the result of cutting can be a single surface or body, which reduces complexity considerably. These conditions are based on congruence and rotational symmetry of the resulting cross sections after cutting and on the knife cutting the origin. Results and applications in the natural sciences will be discussed.

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HYPERSURFACES IN SPACES OF CONSTANT CURVATURE SATISFYING SOME CURVATURE CONDITIONS

MAŁGORZATA GŁOGOWSKA

Abstract. Let g, R, S, κ and C be the metric tensor, the Riemann-Christoffel curvature tensor, the Ricci tensor, the scalar curvature and the Weyl conformal curvature tensor of a semi-Riemannian manifold (M, g), $\dim M \geq 4$, respectively. Let $\mathcal{U}_S$, resp., $\mathcal{U}_C$, be the set of all points of M at which S is proportional to g, resp., the set of all points of M at which C is non-zero. Semi-Riemannian manifolds, and in particular hypersurfaces, satisfying on $\mathcal{U}_S \cap \mathcal{U}_C$ conditions of the form:
$(*)$ the difference tensor $R \cdot C - C \cdot R$ is a linear combination of some $(0, 6)$-tensors formed by symmetric $(0, 2)$-tensors and generalized curvature tensors, were studied in several papers. We refer to [2], [3] and [4] for a survey on manifolds (hypersurfaces) satisfying conditions $(*)$. In this talk we present results on hypersurfaces M isometrically immersed in a semi-Riemannian spaces of constant curvature satisfying on $\mathcal{U}_S \cap \mathcal{U}_C \subset M$ some conditions of the form $(*)$ contained in [1], [5], [6].

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SEMI-SYMMETRIC METRIC CONNECTION AND ITS APPLICATIONS TO FRENET CURVES

ŞABAN GÜVENÇ

Abstract. In a Riemannian manifold (M^n, g), a linear connection $\widetilde{\nabla}$ is called a semi-symmetric metric connection if its torsion tensor field T satisfies

$$T(X,Y) = \omega(Y)X - \omega(X)Y$$

and

$$\left(\widetilde{\nabla}_Z g\right)(X,Y) = 0,$$

where ω is a 1-form and $X, Y, Z \in \chi(M)$. Let ∇ denote the Levi-Civita connection on M. Then, the semi-symmetric metric connection has the form

$$\widetilde{\nabla}_X Y = \nabla_X Y + \omega(Y)X - g(X,Y)U,$$

where U is a vector field associated to ω, i.e. $\omega(X) = g(X,U)$, $\forall X \in \chi(M)$ [4]. In this talk, using a semi-symmetric metric connection, Frenet curves and their frame fields will be reconsidered. We give examples of semi-symmetric Frenet curves in 3-dimensional Eucliean, Sasakian and Kenmotsu manifolds.

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IMPACT OF QUASI-CONSTANT CURVATURE IN $f(\mathcal{R}, G)$ AND $f(\mathcal{R}, \mathcal{T})$-GRAVITY

DIPANKAR HAZRA[1] AND UDAY CHAND DE[2]

Abstract. First, we show that a spacetime of quasi-constant curvature is a static spacetime as well as a generalized Robertson-Walker spacetime under certain restrictions on the associated scalars. As a consequence, we prove that such a spacetime becomes a Robertson-Walker spacetime and is of Petrov type I, D or O. We investigate this spacetime as a solution of $f(\mathcal{R}, G)$-gravity and $f(\mathcal{R}, \mathcal{T})$-gravity theories and explain the physical meaning of the Friedmann-Robertson-Walker metric. For the models $f(\mathcal{R}, G) = 2\mathcal{R} + \lambda G$ (λ is constant) and $f(\mathcal{R}, \mathcal{T}) = \mathcal{R} + 2\mathcal{T}$, various energy conditions in terms of associated scalars are examined. These models satisfy the weak, null, and dominant energy conditions, while violating the strong energy condition, which is in good agreement with recent empirical investigations that show the Universe is currently in an accelerating phase.

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CLASSIFICATION OF SELF-SIMILAR MOTION FOR THREE POINT VORTEX SYSTEM BASED ON DEVIATION CURVATURE

YUMA HIARAKUI[1], TAKAHIRO YAJIMA[2]

Abstract. In this talk, we discuss the behavior of a point vortex system based on deviation curvatures on the Jacobi field. Initially, eigenvalues of the deviation curvatures are represented by relative distances of point vortices in a three-point vortex system. Based on the assumption of self-similar motion for the point vortices, time evolutions of the eigenvalues of the deviation curvatures are shown. When the relative distances change monotonously, the self-similar motions of the three-point vortices are classified into two types, expansion and collapse. Then, it is found that the eigenvalues of the deviation curvature of self-similarity are proportional to the inverse fourth power of the relative distances of the point vortices. The eigenvalues of the deviation curvatures monotonically convergent to zero for the expansion, while they monotonically diverge for the collapse, which expresses that the strengths of interactions between the point vortices relate to the time evolution of a geometric structure given by the deviation curvatures. Notably, in case of the collapse, the collision point becomes a geometric singularity because the eigenvalues of the deviation curvature diverge. These results indicates that the self-similar motions of the point vortices is geometrically classified by the eigenvalues of the deviation curvature.

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REAL HYPERSURFACES IN $S^6(1)$ WITH A CONDITION ON THE STRUCTURE JACOBI OPERATOR

DJORDJE KOCIĆ

Abstract. It is well known that the sphere $S^6(1)$ admits an almost complex structure J which is nearly Kähler. If M is a hypersurface of an almost Hermitian manifold with a unit normal vector field N , the tangent vector field $\xi = -JN$ is said to be characteristic. The Jacobi operator with respect to ξ is called structure Jacobi operator and is denoted by $l = R(\cdot, \xi)\xi$, where R is the curvature tensor on M.
The study of hypersurfaces of almost Hermitian manifolds by means of their Jacobi operators has been highly active in recent years. Specially, many recent results answer the question of the existence of hypersurfaces with a structure Jacobi operator that satisfies conditions related to their parallelism.
We investigate real hypersurfaces in nearly Kähler sphere $S^6(1)$ whose Lie derivative of structure Jacobi operator coincides with the covariant derivative of it, i.e. $\mathcal{L}_X l = \nabla_X l$, $\forall X \in TM$, and show that such submanifolds do not exist.

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NORMALIZED NULL GRAPHS OF GRW SPACETIMES

RAKESH KUMAR

Abstract. To overcome the anomalies caused by the existence of the induced degenerate metric, Duggal and Bejancu [1] fixed the geometric data consisting of a null section and a screen distribution along the null hypersurface. Since the geometric data are chosen arbitrarily and independently, the induced geometric objects by this approach are not always intrinsically connected to the geometry of such hypersurfaces, except in cases of umbilicity or geodesibility. Later, Gutierrez and Olea [2] presented a rigging technique to explore the geometry of null hypersurfaces of a Lorentzian manifold. In this technique, they fixed the geometric data formed by a unique vector field called the 'rigging vector field', which allowed them to construct a 'Riemannian metric' on the null hypersurface. We study normalized null hypersurface given by the graph of a function on the fiber of generalized Robertson-Walker (GRW) spacetime. Using the correspondence between totally umbilical null hypersurfaces of GRW spacetimes and twisted decomposition of the fibres, we compute rigged scalar curvature of totally umbilical null graphs of GRW spacetimes. A special attention is paid to totally geodesic null graphs of GRW spacetimes.

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GEOMETRIC ASPECTS OF CONFORMAL RIEMANNIAN MAPS WHOSE TOTAL MANIFOLD ADMITS A RICCI SOLITON

REETU MAINI

Abstract. We study conformal Riemannian maps between the Riemannian manifolds. We derive conditions for such maps to be harmonic. Later, we study conformal Riemannian maps whose total manifold admits a Ricci soliton and present a non-trivial example of such conformal Riemannian maps. We also obtain conditions for fiber and range space of such maps to be Ricci soliton and Einstein. We derive conditions for conformal Riemannian maps whose total manifold admits a Ricci soliton to be harmonic and biharmonic.

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A NEW CURVATURELIKE TENSOR FIELD IN ALMOST CONTACT MANIFOLDS

KOJI MATSUMOTO

Abstract. Recently, we introduced the new curvaturelike tensor field named (CHR)-curvature tensor which is an almost contact version of Prof. M. Prvanovic ([4]) in almost contact metric manifolds ([1]).

In this talk, as the first step, we give many properties of a trans-Sasakian manifold which is the generalization of a Sasakian and a Kenmotsu manifolds ([2],[3]). Then we mainly consider this tensor field in a trans-Sasakian manifold. Finally, we consider a trans-Sasakian manifold wirh the recurrent Riemannian curvature tensor and the recurrent (CHR)-curvature tensor.

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CHEN AND RICCI INEQUALITIES FOR CERTAIN TYPES OF SUBMANIFOLDS IN THE SEMI-RIEMANNIAN CASE

MARIUS MIREA

Abstract. The isotropic submanifolds of pseudo-Riemannian manifolds are a distinguished class generalizing totally geodesic and totally umbilical ones. They were studied by a number of authors. In the context of isotropic submanifolds we present Chen, and generalized Euler and Ricci inequalities. We also give an example of an isotropic immersion where the equality case of the Chen (first) inequality is obtained.

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RIGIDITY OF HYPERSURFACES WITH CONSTANT HIGHER ORDER MEAN CURVATURE IN SPACE FORMS

OSCAR PALMAS

Abstract. We consider complete oriented hypersurfaces M immersed into a Riemannian space form with constant higher order mean curvature and having two principal curvatures, one of them simple. We impose further restrictions on the Ricci curvature to prove that they be must be isoparametric, generalizing or giving alternate proofs of other characterization results.

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TWO TENSORS OF TYPE (1,2) ASSOCIATED TO THE SHAPE OPERATOR OF A REAL HYPERSURFACE IN THE COMPLEX PROJECTIVE SPACE

JUAN DE DIOS PÉREZ[1], DAVID PÉREZ-LÓPEZ[2]

Abstract. Following S. Tachibana, [2], we study purity and hybridness of two tensors of type (1,2) associated to the shape operator of a real hypersurface in complex projective space, see [1].

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ON CURVATURE OF A MANIFOLD ENDOWED WITH AN ARBITRARY BILINEAR FORM

MILOŠ Z. PETROVIĆ

Abstract. We discuss the question of determining the curvature on a manifold endowed with an arbitrary bilinear form $b = g + \omega$, where some additional requirements are posed on symmetric bilinear form g or skew-symmetric bilinear form ω. We recall known results on the given topic, discuss open problems, and possible further research directions.

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AN INFORMATION GEOMETRY APPROACH TO PATTERN RECOGNITION

IOANA RĂDULESCU (LĂZĂRESCU)

Abstract. Classification is a fundamental task in Machine Learning, where the goal is to assign predefined labels to input data points based on their features. The Support Vector Machine (SVM) is one of the powerful learning techniques for pattern recognition. It embeds patterns into a higher-dimensional space and uses a kernel function to calculate outputs. Computation difficulties caused by large degrees of freedom are avoided when using the kernel method. By analyzing the geometry and the Riemannian structure of the SVM, a method is proposed by Amari in [1] to improve the performance of a kernel.

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ALMOST COMPLEX METALLIC NORDEN MANIFOLDS AND THEIR ADAPTED CONNECTIONS

RACHNA RANI

Abstract. We study almost complex metallic Norden manifolds and their adapted connections with respect to an almost complex metallic Norden structure. We study various connections like special connection of the first type, special connection of the second type, Kobayashi-Nomizu metallic Norden type connection, Yano metallic Norden type connection etc., on almost complex metallic Norden manifolds.

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THE FIRST EIGENVALUE OF THE LAPLACIAN FOR A COMPACT SPACELIKE SUBMANIFOLD IN LORENTZ-MINKOWSKI SPACETIME OF ARBITRARY DIMENSION

ALFONSO ROMERO

Abstract. Eigenvalues of the Laplacian of a compact Riemannian manifold are Riemannian invariants of analytic nature. In particular, the first (non-trivial) eigenvalue contains significant intrinsic information about a compact Riemannian manifold. In the case of a compact submanifold of Euclidean space, the search for relations between the first eigenvalue of the Laplacian of the submanifold and some extrinsic quantity appears naturally. In this direction, R.C. Reilly proved, in his relevant paper [2], that the first eigenvalue of an n-dimensional compact submanifold in an $(n+p)$-dimensional Euclidean space is bounded above by n times the average value of the square of the norm of the corresponding mean curvature vector field, i.e., n times the quotient of the integral on the submanifold of the squared norm of the mean curvature vector field over the volume of the submanifold. Moreover, if the eigenvalue achieves this bound, then the submanifold lies minimally in a hypersphere of the Euclidean space.

Through a counter-example, we will show that Reilly's result does not work for a (codimension ≥ 2) compact spacelike submanifold of Lorentz-Minkowski spacetime [1]. In the search for an alternative result, the original Reilly's proof [2] is recalled, and it is noted that it does not apply to a compact spacelike submanifold of Lorentz-Minkowski spacetime.

Then, the new technique introduced in [1], based on an integral formula on a compact spacelike section of the light cone in Lorentz-Minkowski spacetime, will be developed. The technique is genuine in our setting, that is to say, it cannot be extended to other semi-Euclidean spaces of a higher index. As an application, a family of upper bounds for the first eigenvalue of the Laplacian of a compact spacelike submanifold of Lorentz-Minkowski spacetime is obtained [1]. The equality for one of these inequalities is next geometrically characterized. Indeed, the eigenvalue achieves one of these upper bounds if and only if the compact spacelike submanifold lies minimally in a hypersphere of a certain spacelike hyperplane.

On the way, the Reilly original result is reproved if a compact submanifold of Euclidean space is naturally seen as a compact spacelike submanifold of Lorentz-Minkowski spacetime through a spacelike hyperplane.

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GEOMETRY OF ALMOST NORDEN SUBMERSIONS BETWEEN ALMOST NORDEN MANIFOLDS

RASHMI SACHDEVA

Abstract. We define an almost Norden submersion (holomorphic and semi-Riemannian submersion) between almost Norden manifolds and show that, in most of the cases, the base manifold has the similar kind as that of total manifold. We obtain necessary and sufficient conditions for almost Norden submersion to be a totally geodesic map. We also derive decomposition theorems for the total manifold of such submersions. Moreover, we study the harmonicity of almost Norden submersions between almost Norden manifolds and between Kaehler-Norden manifolds. Finally, we derive conditions for an almost Norden submersion to be a harmonic morphism.

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LIGHTLIKE SUBMERSIONS

BAYRAM ŞAHIN

Abstract. In this talk, the geometry of lightlike submersions in semi-Riemann geometry will be presented. Two separate models will be given in details. Subclasses of these models will also be introduced and examples will be given for each subclass. In addition, for each subclass, O'Neill tensor fields and equations containing linear connections and curvature relations will be given. In the last part of the talk, information will be given about the recent studies on the lightlike submersions in indefinite Hermitian manifolds.

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ON ALMOST NORDEN STATISTICAL MANIFOLD

LEILA SAMEREH[1], ESMAEIL PEYGHAN[1], ION MIHAI[2]

Abstract. We consider a statistical connection ∇ on an almost complex manifold with (pseudo-) Riemannian metric, in particular the Norden metric. We investigate almost Norden (statistical) manifolds under the condition that the almost complex structure J is ∇-recurrent. We provide one example of a complex statistical connection.

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CHARACTERIZATIONS OF *GRW* SPACETIMES ADMITTING RICCI-YAMABE SOLITONS

ARPAN SARDAR[1] AND UDAY CHAND DE[2]

Abstract. The purpose of this study is to determine the conditions under which a generalized Robertson-Walker spacetime to be a perfect fluid spacetime. It is established that Ricci-Yamabe solitons and gradient Ricci-Yamabe solitons in generalized Robertson-Walker spacetimes, and in both cases, it is a perfect fluid spacetime and the Weyl tensor is divergence free.

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RICCI YAMABE SOLITON IN THREE-DIMENSIONAL (ε, δ)-TRANS-SASAKIAN MANIFOLDS

G. S. SHIVAPRASANNA[1], MD. SAMIUL HAQUE[2] AND G. SOMASHEKHARA[3]

Abstract. The objective of the present paper is to carry out Ricci Yamabe soliton in three-dimensional (ε, δ)-trans-Sasakian manifold. We study partially Ricci Pseudosymmetric , Weyl Ricci Pseudosymmetric, projectively flat, Einstein semi-symmetric and ξ-projectively flat (ε, δ)-trans-Sasakian manifold . Further, we obtained conditions for Ricci Yamabe solitons to be shrinking or expanding or steady.

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CURVATURE INEQUALITIES ON SCREEN HOMOTHETIC NORMALIZED NULL HYPERSURFACES OF LORENTZIAN MANIFOLDS

AMRINDER PAL SINGH[1], RAKESH KUMAR[1], CYRIAQUE ATINDOGBE[2] AND VARUN JAIN[3]

Abstract. Since the introduction of the rigging technique, in the version as presented in [3], the study of geometry and physics of null hypersurfaces of Lorentzian manifolds has experienced even more renewed interest. This is mainly due to the improvements bring by this technique, which are of twofold: firstly, the normalization depends only on the choice of a rigging vector field, and secondly, a Riemannian structure coupled with the null-geometry is introduced, which is used to study the geometry of null hypersurfaces when the ambient space has a Lorentzian signature. Therefore, accessing relevant informations from the induced Riemannian structure associated with the normalization, this technique produces insight informations about the null-geometry of the hypersurfaces. In this paper, we study null hypersurfaces of a Lorentzian manifold with a closed normalization for the hypersurface. We derive inequalities involving Ricci tensors, scalar curvature, squared mean curvatures for a null hypersurface with a closed normalization of a Lorentzian space form and for a screen homothetic null hypersurface of a Lorentzian manifold. We also establish a generalized Chen-Ricci inequality and Chen's first inequality for a screen homothetic null hypersurface of a Lorentzian manifold with a closed normalization for the hypersurface.

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COMPRESSIBLE EULER AND MHD EQUATIONS ON RIEMANNIAN MANIFOLDS AND ELLIPTIC-HYPERBOLIC PROPERTY

SIVAGURU SRITHARAN

Abstract. In this talk we will discuss nonlinear mixed (Elliptic-hyperbolic) PDE problem that arises in delta wing aerodynamics at supersonic and hypersonic speeds. By projecting the governing equations to a unit sphere, we arrive at a system of nonlinear PDE which are hyperbolic in part of the sphere and elliptic in another part of the sphere. Moreover, there also shock waves in the flow field. This problem is fundamental to high speed aerodynamics.

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ON RIEMANNIAN MANIFOLDS ADMITTING ζ(RIC)-VECTOR FIELDS

HAKAN METE TAŞTAN[1], MOCTAR TRAORE[1] AND SIBEL GERDAN AYDIN[2]

Abstract. We consider ζ(Ric)-vector fields on Riemannian manifolds. We get some new properties of such vector fields. Then, we investigate the relationships of such vector fields with the well known vector fields in the literature. Also, we consider and study these vector fields under some topological restrictions of the manifolds on which they are defined. We prove that any contractible Ricci soliton whose potential vector field ζ(Ric) is gradient. Moreover, we apply such vector fields to Ricci solitons. The soliton is either steady or it is Einsteinian and locally a warped product. Finally, using the Hodge-de Rham decomposition theorem, we show that if the potential vector field of a compact, orientable and without boundary Ricci soliton is ζ(Ric), then the soliton is steady.

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SOME CHARACTERIZATIONS ON GRADIENT ALMOST η-RICCI-BOURGUIGNON SOLITONS

MOCTAR TRAORE, HAKAN METE TAŞTAN, AND SIBEL GERDAN AYDIN

Abstract. In this study, we investigate Riemannian manifold with gradient almost η-Ricci Bourguignon solitons structures. Then we show that a gradient almost η-Ricci-Bourguignon soliton is gradient $(-\frac{1}{\omega u})$-almost traceless Ricci soliton with the potential function k. Moreover, we prove that a gradient $(-\frac{1}{\omega u})$-almost traceless Ricci soliton is isometric to a standard unit sphere $\mathbb{S}^n$, hyperbolic space $\mathbb{H}^n$ and Euclidean space $\mathbb{R}^n$ with constant scalar curvature or its associated vector fields is conformal. Finally, we deduce some properties of integral formula for the gradient compact case.

Keywords: Gradient η-Ricci-Bourguignon soliton, Conformal vector field, traceless Ricci soliton, space form.
2010 Mathematics Subject Classifcation: 53C25, 53C21, 53E20.

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COMPREHENSIVE REAL SPACE FORM

MUKUT MANI TRIPATHI

Abstract. The definitions of a comprehensive real space form and a comprehensive Einstein manifold are introduced. All known particular cases are discussed in detail. Several problems are proposed at the end. The talk is part of a project "A Manual of Different Kinds of Space Forms".

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HEMI-SLANT SUBMERSIONS FROM LOCALLY CONFORMAL KAEHLER MANIFOLDS

DENIZ ULUSOY, HAKAN METE TAŞTAN

Abstract. We study hemi-slant submersions from locally conformal Kaehler manifolds. We first give conditions for distributions which are involved in the definition of the submersion to be integrable. We investigate the harmonicity of such submersion. Moreover, we give a necessary and sufficient condition for these types of submersions to be Clairaut.

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MINIMAL LAGRANGIAN SUBMANIFOLDS OF COMPLEX (HYPERBOLIC) QUADRICS

JOERI VAN DER VEKEN

Abstract. The complex quadric and its non-compact dual, the complex hyperbolic quadric, are Kähler-Einstein spaces that exist in any complex dimension. In this work, we study minimal Lagrangian submanifolds of these spaces and give classifications under additional assumptions. A key observation is that these Lagrangian immersions can be seen as Gauss maps of hypersurfaces of spheres and of spacelike hypersurfaces of anti-de Sitter spaces respectively. All this can be seen as a wide generalization of work of I. Castro and F. Urbano on minimal Lagrangian surfaces in $S^2 \times S^2$ [Comm. Anal. Geom. 15 (2007), 217-248], since this ambient space is precisely the complex two-dimensional quadric.

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FINSLER SPACETIMES WITH (α, β)-METRICS AND THEIR ISOMETRIES

N. VOICU[1], E. POPOVICI-POPESCU[1], A. FRIEDL-SZASZ[1] AND C. PFEIFER[2]

Abstract. The class of (α, β)-metrics is the most common and most used subclass of Finsler metrics, obtained by deforming a given pseudo-Riemannian metric α by means of a 1-form β. After briefly discussing the physical motivations for studying this class, we establish the conditions such that an (α, β)-deformation of a Lorentzian metric α admits a well defined causal structure. Moreover, we completely determine the relation between the Killing vector fields of a generic (α, β)-metric and those of the underlying Lorentzian metric α. In particular, we find that there exist (α, β)-metrics which admit symmetries that are not symmetries of α and discuss potential physical applications of this peculiarity.

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AN EXTRINSIC AVERAGE VARIATIONAL METHOD $\Phi_{(3)}$-HARMONIC MAPS AND $\Phi_{(3)}$-SSU MANIFOLDS

SHIHSHU WALTER WEI

Abstract. Observing Mathematics and Nature are beautifully interwoven, and are frequently two sides of the same coin, We proposed an extrinsic, average variational method (cf. [1, 2]) as an approach to confront and resolve problems in global, nonlinear analysis, geometry and physics. Well discuss $\Phi_{(3)}$-harmonic maps and new type of manifolds $\Phi_{(3)}$-SSU manifolds found by this method [3].

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