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BOOK

OF

ABSTRACTS

Riemannian Geometry and Applications – RIGA 2025
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SLANT SUBMANIFOLDS OF PARA-COMPLEX AND NORDEN MANIFOLDS

P. ALEGRE, A. CARRIAZO, F. PLANAS.

Abstract. In [5], B.-Y. Chen introduced slant submanifolds of an almost Hermitian manifold, (M, J, g) as those submanifolds for which the angle θ between JX and the tangent space is constant, for any tangent vector field X. They play an intermediate role between complex submanifolds ($\theta = 0$) and totally real ones ($\theta = \pi/2$). Since then, the study of slant submanifolds has produced an incredible amount of results and examples. Moreover, some generalizations of them have also been defined, such as semi-slant, bi-slant or generic submanifolds. Also the study of slant submanifolds has been extended both to odd dimensional and semi-Riemannian environments.

Consider a (M, J, g) with J a $(1, 1)$ tensor field such as $J^2 = \pm I$ and g a metric, it could be considered different cases depending on the compatiblity relatioship existing between both J and g:

	$g(JX, JY) = g(X, Y)$	$g(JX, JY) = -g(X, Y)$
$J^2 = -I$	almost complex manifold	Norden manifold
$J^2 = I$	almost product manifold	almost para-complex manifold

These ambient spaces have a rich structure, similar to that of almost Hermitian ones, but also with very interesting differences. Norden manifolds were firstly called B-manifolds by A.P. Norden [6] and sometimes they are also known as anti-Kaehler [4].

Slant submanifolds of an almost product manifold were introduced by B. Sahin [7], the first two authors studied curves as slant submanifolds, [2], and innitiated the study of slant submanifolds of a almost para-complex manifold, [1]. The objetive of the present talk is fullfill this panorama defining slant submanifolds of a Norden manifold, and highlight some differences and similarities in the different ambient spaces.

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TORQUED AND ANTI-TORQUED VECTOR FIELDS ON SPACE FORMS

MUHITTIN EVREN AYDIN[1], ADELA MIHAI[2], CIHAN ÖZGÜR[3]

Abstract. In this talk, we aim to characterize the complete, simply connected models of Riemannian manifolds with constant sectional curvature using special vector fields, specifically the torqued and anti-torqued vector fields. In particular, we focus on the problem of determining the existence of such vector fields in the hyperbolic space $\mathbb{H}^n$, providing non-trivial examples.

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CLASSIFICATION OF SPACE-LIKE AND TIME-LIKE SURFACES IN WARPED PRODUCT SPACE-TIMES WITH POSITIVE RELATIVE NULLITY

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Abstract. Let Σ be a space-like or time-like surface in a Lorentzian manifold. The relative null space at a point $p \in \Sigma$, denoted N_p, is defined as:

$$N_p = \{X_p \in T_p\Sigma \mid h_p(X_p, Y_p) = 0 \text{ for all } Y_p \in T_p\Sigma\},$$

where h_p is the second fundamental form at p, and $T_p\Sigma$ is the tangent space of the surface at p. If the dimension of the relative null space is positive at every point on Σ, then the surface Σ is said to have positive relative nullity.

This talk focuses on space-like and time-like surfaces with positive relative nullity in warped product space-times, specifically within the framework of Robertson–Walker space-times denoted by $L_1^4(f, c)$. The necessary and sufficient conditions for the existence of such surfaces are presented, along with the forms of their Levi-Civita connections. Local classification theorems for space-like and time-like surfaces in the special case $L_1^4(f, 0)$ are also discussed. The talk concludes with results concerning space-like and time-like surfaces in the warped product spaces $\mathbb{E}_1^1 \times \mathbb{S}^3$ and $\mathbb{E}_1^1 \times \mathbb{H}^3$, which correspond to particular cases of $L_1^4(f, c)$ when the warping function is constant, with $c = 1$ for $\mathbb{E}_1^1 \times \mathbb{S}^3$ and $c = -1$ for $\mathbb{E}_1^1 \times \mathbb{H}^3$.

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GEOMETRIC PROPERTIES OF A HYPERSURFACE IN THURSTON GEOMETRY Nil^4

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Abstract. After a brief review of classification of four dimension Thurston geometries, we study the geometric properties of a hypersurface in Thurston geometry Nil^4, namely parallel hypersurface, Codazzi hypersurface and pseudo parallel hypersurface.

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A FAMILY OF TIMELESS SIX-DIMENSIONAL UNIVERSES

WLADIMIR-GEORGES BOSKOFF

Abstract. The talk highlights a family of timeless universes connected by the same simple change of coordinates which allows to transfer between the two metrics involved in the description of each such universe. The first example can be called the simple-basic timeless universe. This universe allows one to create some other timeless universes, one of them being without matter, one in expansion and another one of the wormhole type. In all these universes, there are regions where classical physical time exists, but a global time coordinate indicating the temporal order of events does not exist. This happens because, in these universes, we can mathematically construct future oriented time-like loops and closed chains of future oriented time-like curves. The example of timeless expanding universe shows that time, in general, is not a consequence of space expansion.

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RECTIFYING CURVES AND THEIR INFINITESIMAL BENDINGS

STEFAN-CEZAR BROSCATEANU

Abstract. Both notions, of an infinitesimal bending of a curve and of a rectifying curve, play important roles in the theory of curves. We begin the study of the infinitesimal bending of a rectifying curve for some particular cases. Geometrical interpretations are given. A conjecture is formulated and new research directions are suggested.

This is a joint work with Adela Mihai and Andreea Olteanu.

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EXPLORING BROWNIAN MOTION ON DIVERSE STRUCTURES

OVIDIU CALIN

ABSTRACT. In this talk, we delve into the captivating interplay between stochastic processes and geometric structures, with a focus on Brownian motion as it unfolds across lines, circles, curves, and planar domains. We investigate the intricate probability laws governing these motions and reveal surprising phenomena that arise when randomness meets geometry.

A highlight of the presentation is the study of the so-called Brownian trianglea triangle whose vertices are determined by three independent planar Brownian motions. We analyze the distribution of its area, uncovering remarkable stochastic properties and connections to classical geometric probability.

In addition, the talk addresses the relationship between curvature and randomness. We explore how the curvature of a curve influences the behavior of a Brownian particle traveling near it over small time intervals, shedding new light on the interplay between deterministic geometry and probabilistic dynamics.

Attendees will gain insight into modern techniques in stochastic geometry and see how these methods open doors to novel applications and theoretical advancements in the understanding of complex random systems.

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QUADRATIC ROTATIONAL WEINGARTEN SURFACES

PAULA CARRETERO[1], ILDEFONSO CASTRO[2], ILDEFONSO CASTRO-INFANTES[3]

ABSTRACT. The classification of rotational surfaces in Euclidean space satisfying a linear relation between their principal curvatures was completed in [1]. On the other hand, using the notion of geometric linear momentum of a planar curve with respect to a line introduced in [2], a new approach to rotational Weingarten surfaces was developed in [3]. Taking advantage of this study, we face the case that the principal curvatures satisfy a certain quadratic relation.

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A NEW INSIGHT ON ROTATIONAL SURFACES IN LORENTZ-MINKOWSKI SPACE

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Abstract. The notions of geometric linear momentum of a Euclidean or Lorentzian planar curve with respect to a line were introduced in [2], [3] and [4]. Using them as a key tool, we make a new approach to the rotational surfaces in Lorentz-Minkowski space along the sames lines as [1] through the corresponding geometric linear momenta of the generatrix curve with respect to the axis of revolution. In this way, we classify certain families of rotational Weingarten surfaces.

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DISAFFINITY VECTOR FIELDS ON RIEMANNIAN MANIFOLDS AND THEIR APPLICATIONS

BANG-YEN CHEN

ABSTRACT. First, I review in this talk several well-known vector fields on a Riemannian manifold. Then, I define disaffinity vector fields on Riemannian manifolds. After that, I present some relationships between disaffinity vector fields, Killing vector fields, and incompressible vector fields on Riemannian manifolds. Finally, I provide several applications of disaffinity vector fields to soliton theory.

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A NOTE ON SCHOUTEN AND COTTON TENSORS IN SPACE-TIMES

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ABSTRACT. In the literature, many researchers have studied various tensor fields from different perspectives. Among these, Schouten and Cotton tensor fields are discussed in this study. Firstly, basic information about the algebraic type of the Schouten tensor in space-times is presented by considering the classification of such kind of tensor fields. Regarding the Schouten tensor's eigenvalue-eigenvector structure, certain links are identified in case the manifold contains some special vector fields. Afterwards, in the light of previously obtained results, studies are made on the Cotton tensor. Examples on space-times related to the outcomes are constructed and the existence of the aforementioned tensor fields in holonomy theory is investigated.

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COMPACT SPACELIKE SUBMANIFOLDS IN PSEUDO-RIEMANNIAN SPACE FORMS OF NON-NEGATIVE SECTIONAL CURVATURE

ALEXANDRU CIOBANU

We investigate compact spacelike submanifolds M^n immersed in a pseudo-Riemannian space form $N_p^{n+p}(c)$, particularly in the pseudo-Euclidean space $\mathbb{R}_p^{n+p}$ and the de Sitter space $M_p^{n+p}(c)$, assuming the flatness of the normal bundle. Under some intrinsic conditions on M^n, we give a classification in $N_p^{n+p}(c)$. We also present examples of submanifolds, sustaining the statements of the theorems.

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COSMOLOGICAL LANDSBERG FINSLER SPACETIMES

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Abstract. We classify all possible cosmological homogeneous and isotropic Landsberg-type Finsler structures, in 4-dimensions. Among them, we identify viable non-stationary Finsler spacetimes, i.e. those geometries leading to a physical causal structure and a dynamical universe. Noting that any non-stationary and non-Riemannian Landsberg metric must be actually also non-Berwald (i.e., it should be a so-called *unicorn*), we construct the *unique* Finsler, non-Berwaldian Landsberg generalization of Friedmann-Lemaitre-Robertson-Walker geometry.

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INEQUALITY AND DIVERSITY: INSIGHTS FROM BIOLOGY

JOHAN GIELIS

ABSTRACT. Inequality is an important topic in economics and human society, and various models have been proposed based on Lorenz curves and Gini coefficients. Models in biology, at the level of individual beings, organizations, or ecosystems, are very similar.

To assess the inequality of plant leaf size on plants [1], the dynamics of fruit ripening in individual plants [2], or the diversity of tree size in forests [3], (generalized) performance equations have been tested. Using Shi-rotations [1],[4], it turns out that the performance equations of biology are the Lorenz curves of economics [5].

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GEOMETRIC INSIGHTS OF SUBMERSION BETWEEN NORDEN MANIFOLDS

GARIMA GOEL

ABSTRACT. We define an almost Norden submersion (holomorphic and semi-Riemannian submersion) between almost Norden manifolds and show that, in most of the cases, the base manifold has the similar kind of structure as that of total manifold. We obtain necessary and sufficient conditions for almost Norden submersion to be a totally geodesic map. We also derive decomposition theorems for the total manifold of such submersions. Moreover, we study the harmonicity of almost Norden submersions between almost Norden manifolds and between Kaehler-Norden manifolds. Finally, we derive conditions for an almost Norden submersion to be a harmonic morphism.

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ON A CERTAIN CLASS OF ROTATIONAL HYPERSURFACES SATISFYING $\Delta \mathbf{x} = \mathcal{A}\mathbf{x}$ IN THE SIX-DIMENSIONAL EUCLIDEAN SPACE

ERHAN GÜLER*, YUSUF YAYLI, AND HASAN HILMI HACISALIHOĞLU

Abstract. In this paper, a specific class of rotational hypersurfaces $\mathbf{x}$ with five parameters in the six-dimensional Euclidean space $\mathbb{E}^6$ is investigated. The i-th curvature functions associated with these hypersurfaces are explicitly computed, and their geometric properties are examined. Furthermore, the action of the Laplace–Beltrami operator on such hypersurfaces is analyzed, and the conditions under which the relation $\Delta \mathbf{x} = \mathcal{A}\mathbf{x}$ holds for a 6×6 matrix $\mathcal{A}$ are determined.

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A STUDY ON FOUR-DIMENSIONAL RICCI SOLITONS

BAHAR KIRIK RÁCZ[1], BURCU CINDIK[2,3]

Abstract. The aim of this work is to examine Ricci solitons, which are among the popular research topics of differential geometry nowadays, by focusing on the 4−dimensional case. In this respect, 4−dimensional manifolds admitting different metric signatures are considered by expanding the studies in the literature. Several examples of Ricci solitons are given for 4−dimensional manifolds with Lorentz or neutral metric signatures. In the case of the manifold containing a parallel vector field, its relationship with the potential field is investigated and steady Ricci solitons are obtained under certain conditions. A similar analysis is made for recurrent vector fields. More explicitly, an example of a Ricci soliton whose potential field is recurrent is found and the Segre type of the Ricci tensor is determined. The results in question are interpreted by associating them with concepts such as holonomy theory and another vector fields on the relevant manifolds.

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ON THE GEOMETRY OF SPACELIKE HYPERSURFACES OF THE GENERALIZED ROBERTSON-WALER SPACETIMES

AMRINDER PAL SINGH[1], ALEXANDRU CIOBANU[2], RAKESH KUMAR[1⋆], ADELA MIHAI[3], MUHITTIN EVREN AYDIN[4]

Abstract. We explore a novel approach by considering the fiber of the Generalized Robertson-Waler (GRW) spacetime as a statistical manifold. This perspective allows us to endow the GRW spacetime with a statistical structure. We derive the necessary and sufficient conditions for the existence of a statistical Ricci soliton on the statistical spacelike hypersurface of GRW spacetime. We explore the scenario where the fiber of the GRW spacetime constitutes a statistical Ricci soliton, leading to the derivation of conditions under which the GRW spacetime itself forms an almost Ricci soliton. Furthermore, we examine the case where both the fiber and the GRW spacetime are Ricci solitons, and we establish the necessary and sufficient conditions for the statistical spacelike hypersurface to exhibit a Ricci soliton structure.

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FUNDAMENTALS OF NEARLY KÄHLER $\mathbb{C}P^3$: CURVATURE, ISOMETRIES, AND HYPERSURFACES

MICHAËL LIEFSOENS

ABSTRACT. This talk will cover the fundamental geometric structures of the nearly Kähler $\mathbb{C}P^3$, and the role this manifold plays in the general theory of nearly Kähler manifolds. Moreover, we will discuss the curvature tensor and isometry groups for all metrics g such that $(\mathbb{C}P^3, g)$ is homogeneous. Finally, some hypersurfaces of the nearly Kähler $\mathbb{C}P^3$ are studied and classified.

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NATURAL CONNECTIONS ON RIEMANNIAN Π-MANIFOLDS

HRISTO MANEV

ABSTRACT. The study of the differential geometry of Riemannian Π-manifolds, also known as almost paracontact almost paracomplex Riemannian manifolds, is continued. Natural connections on these manifolds are investigated.

A natural connection with torsion is defined and it is called the first natural connection on Riemannian Π-manifold. Relations between the introduced connection and the Levi-Civita connection are obtained, as well as relations between their respective curvature tensors, torsion tensors, Ricci tensors, and scalar curvatures in the main classes of a classification of Riemannian Π-manifolds are presented.

Another natural connection, determined by a property of its torsion tensor, is defined and it is called the second natural connection on Riemannian Π-manifold. The uniqueness of this connection is proved and a necessary and sufficient condition for coincidence with the first natural connection on the considered manifolds is found. The form of the torsion tensor of the second natural connection is obtained in the classes of the Riemannian Π-manifolds in which it differs from the first natural connection. All the main classes of considered manifolds are characterized with respect to the torsion of the second natural connection.

Moreover, a natural connection whose torsion tensor is a 3-form is defined and called the third natural connection on a Riemannian Π-manifold. In the cases of existence of this connection, a relation between it and the first and second natural connections on the considered manifolds is established.

Explicit examples of dimension 5 are given in support of the proven assertions.

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RICCI–BOURGUIGNON ALMOST SOLITONS WITH VERTICAL TORSE-FORMING POTENTIAL ON ALMOST CONTACT COMPLEX RIEMANNIAN MANIFOLDS

MANCHO MANEV

ABSTRACT. In the present work, we equip almost contact complex Riemannian manifolds (abbreviated accR manifolds), also known as almost contact B-metric manifolds (see [1]), with Ricci–Bourguignon-like almost solitons. In [2] we started the study of Ricci–Bourguignon almost solitons on accR manifolds, exploiting the presence of a pair of metrics related to each other through the structure of the studied manifolds. We have generalized the known Ricci–Bourguignon almost solitons so that in addition to the main metric, the associated metric of the manifold is also used. In the present work, we specialize the soliton potential to be pointwise collinear with the Reeb vector field of the manifold structure, as well as torse-forming with respect to the two Levi-Civita connections of the pair of B-metrics. We focus on accR manifolds of the basic class $\mathcal{F}_5$ in the classification of [1], since this class is the counterpart of the class of well-known β-Kenmotsu manifolds among almost contact metric manifolds. We find the forms of the Ricci tensor and the scalar curvatures generated by the pair of B-metrics on the studied manifolds with the additional structures. In the three-dimensional case, we construct an explicit example and illustrate some of the properties obtained in the theoretical part.

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RIEMANNIAN INVARIANTS OF SUBMANIFOLDS

SIMONA MARINESCU (DECU)

ABSTRACT. In this talk, we study new Chen inequalities in statistical submanifolds of Kenmotsu statistical manifolds of constant ϕ-sectional curvature. Moreover, we investigate the equality cases of these inequalities. We also reveal an illustrative example.

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A RECURRENT TRANS-SASAKIAN MANIFOLD

KOJI MATSUMOTO

ABSTRACT. A Riemannian manifold is said to be *recurrent* if the Riemannian curvature tensor R is recurrent, that is,

$$(\nabla_V R)(X, Y, Z, W) = \Pi(V)R(X, Y, Z, W),$$

for a certain 1-form Π which is called the *recurrent form*.

In this talk, we consider a recurrent trans-Sasakian manifold. We show that the recurrent form is given by associated functions of the trans-Sasakian manifold. Using this result, we show that the Riemannian curvature tensor in a recurrent trans-Sasakian manifold is given by associated functions.

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TIMELIKE MERIDIAN SURFACES IN THE MINKOWSKI 4-SPACE

VELICHKA MILOUSHEVA

Abstract. We consider a special family of 2-dimensional timelike surfaces in the Minkowski 4-space $\mathbb{R}^4_1$ which lie on rotational hypersurfaces with timelike axis and call them meridian surfaces of elliptic type. We study the following basic classes of timelike meridian surfaces of elliptic type: with constant Gauss curvature, with constant mean curvature, with parallel mean curvature vector field, with parallel normalized mean curvature vector field. The results obtained for the last class are used to give explicit solutions to the background systems of natural PDEs describing the timelike surfaces with parallel normalized mean curvature vector field in $\mathbb{R}^4_1$.

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HOMOGENEOUS MAGNETIC GEODESICS

MARIAN IOAN MUNTEANU

Abstract. Magnetic geodesics are solutions of a second order differential equation (known as the Lorentz equation) and they generalise geodesics. A magnetic field on a Riemannian manifold is defined by a closed 2-form that helps, together with the metric, to define the Lorentz force. On the other hand, magnetic curves derive from the variational problem of the Landau-Hall functional, which is, in the absence of a magnetic field, nothing but the kinetic energy functional.

The challenge is to solve the differential equation in order to find explicit solution, meaning the explicit parametrization for the magnetic trajectories. Nevertheless, this is not always possible and, because of that, it is necessary to understand the behaviour of the solution.

Recent studies are done in 3-dimensional Sasakian manifolds, where the contact 2-form naturally defines a magnetic field. The solutions of the Lorentz equation, are usually called *contact magnetic trajectories*. It can be proved a reduction result for the codimension in a Sasakian space form, that is, essentially, we can reduce the study (of a magnetic curves in a Sasakian space form) to dimension 3. The geometry of magnetic trajectories have been recently studied in the 3-sphere, in the Berger 3-sphere, in the Heisenberg group Nil_3 and in $\mathrm{SL}(2,\mathbb{R})$, respectively.

Conditions for the periodicity of contact magnetic geodesics have been obtained for the unit 3-sphere, Berger 3-sphere and $\mathrm{SL}(2,\mathbb{R})$. It was proved that periodic contact magnetic trajectories can be quantized in the set of rational numbers.

The geometry of contact magnetic curves in $\mathrm{SL}(2,\mathbb{R})$ is much more beautiful. More precisely, it can be shown that every contact magnetic trajectory (of charge q) starting at the origin of $\mathrm{SL}(2,\mathbb{R})$ with initial velocity X and with charge q is the product of the homogeneous geodesic with initial velocity X and the charged Reeb flow $\exp(2qs\xi)$.

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A REVIEW ON MAGNETIC CURVES

ANA-IRINA NISTOR

Abstract. In this talk we summarize the main classification results we obtained in the study of magnetic curves. We recall first the definition of magnetic curves in almost contact metric manifolds and then we develop the study in the Sasakian and the cosymplectic manifolds. Second, we turn our attention on magnetic curves in quasi-Sasakian manifolds. More precisely, we show that the magnetic curves in quasi-Sasakian manifolds, obtained as the product of a Sasakian and a Kähler manifold, have maximum order 5. Finally, we conclude with some examples.

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CHARACTERIZATIONS OF BIHARMONIC CURVES IN WARPED PRODUCT MANIFOLDS

CIHAN ÖZGÜR, ŞABAN GÜVENÇ

Abstract. We consider biharmonic curves in warped product manifolds $I \times_f M^n(c)$, where I is an open interval and $M^n(c)$ is a space of constant curvature. We obtain curvature characterizations of biharmonic curves by four distinct cases. We also give some examples of such curves.

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ON SECTIONAL CURVATURE OF GENERALIZED RIEMANNIAN SPACES IN EISENHART'S SENSE

MILOŠ Z. PETROVIĆ

Abstract. Recall a definition of a generalized Riemannian space in Eisenhart's sense as a differentiable manifold M endowed with a bilinear form $b = g + \omega$, where g is a symmetric non-degenerate bilinear form and ω is a skew-symmetric bilinear form. These spaces were introduced by Eisenhart in 1951, however, up to our knowledge it is not clear how to properly define a sectional curvature which mostly describe the geometry of these spaces. We study various ways of determing the sectional curvature of generalized Riemannian spaces in Eisenhart's sense.

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THE FINSLER FRIEDMANN EQUATION - FROM KINETIC GASSES TO AN EXPONENTIALLY EXPANDING UNIVERSE

CHRISTIAN PFEIFER[1], NICOLETA VOICU[2], ANNAMÁRIA FRIEDL-SZÁSZ[2], ELENA POPOVICI-POPESCU[2]

Abstract. We derive the Finsler gravity equation in homogeneous and isiotropic symmetry in conformal time, the *Finsler Friedmann equation*, and find that it naturally permits an exponential expanding universe, without the need for a cosmological constant or any additional quantities. This solution possesses a causal structure which is a mild deformation of the causal structure of Friedmann-Lemaitre-Robertson-Walker (FLRW) geometry; close to the rest frame defined by cosmological time (i.e. for slowly moving objects) the causal structures of the two geometries are nearly indistinguishable.

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ON THE PRECISE CHARACTERIZATION OF THE CONTACT CR δ-INVARIANT FOR CANONICAL SUBMANIFOLDS IN LORENTZ-SASAKIAN MANIFOLDS

OCTAVIAN POSTĂVARU

Abstract. The notion of contact CR-submanifolds in Sasakian geometry, rooted in classical CR structures arising from real hypersurfaces in complex manifolds, has evolved significantly since its inception. Originally defined in the context of anti-holomorphic submanifolds of complex space forms, the CR δ-invariant has been extended to various ambient geometries, including Sasakian space forms. In this work, we derive a sharp inequality involving the contact CR δ-invariant for contact CR-submanifolds situated in Lorentz-Sasakian space forms, contributing to the broader understanding of curvature relations in semi-Riemannian contact geometry.

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STUDY OF CONFORMAL RIEMANNIAN MAPS WHOSE TOTAL MANIFOLD ADMITS A RICCI SOLITON

RACHNA RANI

ABSTRACT. We study conformal Riemannian maps between the Riemannian manifolds. We derive a Bochner type identity and conditions for such maps to be harmonic. Later, we study conformal Riemannian maps whose total manifold admits a Ricci soliton and present a nontrivial example of such conformal Riemannian maps. We also obtain conditions for fiber and range space of such maps to be Ricci soliton and Einstein. We derive conditions for conformal Riemannian maps whose total manifold admits a Ricci soliton to be harmonic and biharmonic.

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NATURAL AND EUCLIDEAN GRADIENTS IN THE MULTINOMIAL STATISTICAL MANIFOLD

IOANA RĂDULESCU (LĂZĂRESCU)

ABSTRACT. This paper explores the duality of the Natural Gradient and Euclidean Gradient in the statistical manifold of multinomial distributions by examining the relationship between these gradients in dual coordinate spaces. We derive the canonical exponential form of the multinomial distribution and compute the Fisher metric using a change of basis method. The duality between the Natural and Euclidean Gradients in these spaces is demonstrated through both computational derivations and experimental validation. In a small-scale experiment, we compare the convergence rates of these gradients, confirming their duality and highlighting the practical advantages of the natural gradient in optimization.

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GEOMETRIC ANALYSIS OF CONFORMAL RIEMANNIAN MAPS WITH YAMABE SOLITON STRUCTURE

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Abstract. We explore conformal Riemannnian maps between Riemannian manifolds and derive expression of scalar curvature for its total manifold. Later, we study conformal Riemannian maps whose source manifold admits a Yamabe soliton and obtain conditions for fiber and range space of such maps to be Yamabe soliton. We also present a characterization theorem for a Yamabe soliton to be an almost Yamabe soliton for conformal Riemannian maps. Finally, we derive a non-trivial example of a conformal Riemannnian map whose total manifold admits a Yamabe soliton.

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FUNDAMENTAL GAP ESTIMATE ON RIEMANNIAN MANIFOLDS

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ABSTRACT. The fundamental gap is the difference between the first and second eigenvalue of the Laplacian. In this talk, we give an overview of the historical development on research into this gap on Euclidean space and on Riemannian manifolds. In particular, we fill focus on the lower bound estimates of this gap in different settings.

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EXTRINSIC CHARACTERISATIONS OF IMMERSIONS

BOGDAN SUCEAVĂ

Abstract. Leonhard Euler believed that one can not define a good curvature measure for surfaces. He wrote in 1763: "la question sur la courbure des surfaces n'est pas susceptible d'une réponse simple, mais elle exige à la fois une infinité de déterminations." The quest for the right measure for curvature was settled by C.F. Gauss in 1825, and Sophie Germain introduced the mean curvature in 1831 (her memoir written in 1816 included also the average of principal curvature as a shape invariant). This is the origin of an interesting quest: how do we best quantify the deformation of space? The inquiry was pursued well into the 20th century and it is still of interest today. In 1956 John F. Nash, Jr. proved that a Riemannian manifold can be immersed isometrically into an Euclidean ambient space of dimension sufficiently large. In 1968, S.-S. Chern pointed out that a key technical element in applying Nash's Theorem effectively is finding useful relationships between intrinsic and extrinsic quantities characterising immersions. And such relations are few, at least few enough to present us with a technical challenge in applying Nash's Theorem. One technical reason is presented by the passing through the narrow gateway of Gauss' equation, and that's why it is useful to add to this argument other techniques. A turning point in the history of this question was an enlightening paper written by B.-Y. Chen in 1993, which paved the way for a deeper understanding of the meaning of the Riemannian inequalities between intrinsic and extrinsic quantities. Our present discussion invites a reflection on whether we could hope to characterise submanifolds by using mainly extrinsic quantities. What could we hope to obtain?

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ON φ-INVARIANT SASAKI-LIKE STATISTICAL SUBMERSIONS

KAZUHIKO TAKANO

ABSTRACT. Riemann submersion was defined by B. O'Neill (1966) and A. Gray (1967), and since then, there have been many studies, such as the case where the total space has geometric structures such as almost complex structures and almost contact structures.

In 1945, C. R. Rao showed that statistical models can be studied geometrically using the Fisher information as the Riemann metric. Subsequently, in 1985, S. Amari developed statistical manifolds and information geometry based on the dual connection theory. We defined geometric structures similar to almost complex structures (2004) and almost contact structures (2006) in statistical manifolds.

The statistical submersion defined by N. Abe and K. Hasegawa (2001). We studied the statistical submersion in which the total space permits geometric structures similar to almost complex structures or almost contact structures. The even dimensional Poincaré upper half-space does not have a Kählerian structure, but it has a geometric structure similar to that of the Kählerian structure. We constructed this geometric structure from the α-connections of the statistical model of a special multivariate normal distribution (2008).

In this time, we introduce the statistical submersion in which each fiber has a structure similar to the φ-invariant Sasakian structure.

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GENERALIZED WILLMORE SURFACES, GEOMETRIC FLOWS AND APPLICATIONS TO BIOPHYSICS

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Abstract. Functionals involving surface curvature represent an extensive field of study across many sciences, including but not limited to mathematics, mechanics, physics and biology. Their extrema are crucial in the field of biophysics and have numerous applications to the shapes of biological cells, bio-membranes and protein folding. The talk will discuss Willmore energies and generalizations (in particular, p-Willmore energies and Helfrich-Canham energies). Some recent mathematical results of our team of researchers will be presented along with real-life applications and computational models.

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A RIEMANNIAN GEOMETRIC APPROACH TO THE RAYLEIGH DISTRIBUTION

INAN ÜNAL AND MERVE YAPRAK ONAT

Abstract. In this study, we investigate the Rayleigh distribution from the viewpoint of Riemannian geometry. By treating the family of Rayleigh distributions as a one-dimensional statistical manifold, we derive the Fisher information metric and compute the corresponding Christoffel symbols. This allows us to describe the geometric structure of the model and to solve the geodesic equations explicitly. We also compare the Rayleigh model with the exponential distribution and show that they are geometrically equivalent under a smooth reparameterization. The results provide a clearer understanding of the intrinsic geometry and statistical behavior of the Rayleigh distribution.

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METRIZABILITY OF SYMMETRIC AFFINE CONNECTIONS: RIEMANN VERSUS FINSLER

NICOLETA VOICU

ABSTRACT. For a torsion-free affine connection on a given manifold, which does not necessarily arise as the Levi-Civita connection of any pseudo-Riemannian metric, it is still possible that it corresponds in a canonical way to a Finsler structure. This property, known as pseudo-Finsler (or Berwald-Finsler) metrizability, is of core importance for physical applications, as it is equivalent to the local variationality of the equations of the autoparallels of the connection. After introducing some general criteria for Finsler metrizability, we analyze the case of 4-dimensional $SO(3)$-invariant connections. For this class, we are able to determine all symmetric connections which are not Levi-Civita connections for any pseudo-Riemannian metric-hence, are non-metric in a conventional way-but can still be metrized by $SO(3)$-invariant Finsler functions.

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